Zum 1. Teil (S. 381)
Zum Inhaltsverzeichnis

38. DE SERIERUM SUMMIS ET DE QUADRATURIS PLAGULAE QUINDECIM

Oktober 1674 und [Anfang 1678 – Ende 1679]

Die folgenden Stücke sind von Leibniz zu einer Gruppe zusammengefaßt worden, in der er bogenweise durch Überschriften fünfzehn (recte: sechzehn) Teile unterscheidet; die Überschrift Pars quarta erscheint doppelt. Der letzte Bogen enthält zusätzlich ein Einlegeblatt und trägt keine Überschrift. Die ersten drei Bögen sind mit dem Datum Oktober 1674 versehen.

In Hannover hat Leibniz einen Umschlag hinzugefügt, der eine erweiterte Überschrift und das Datum Oktober 1674 der Stücke trägt. Das Wasserzeichen des Umschlagbogens ist für die Jahre 1678/1679 belegt.

38_1. UMSCHLAG

[Anfang 1678 – Ende 1679]

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 1+37. 1 Bog. 2°. Hannoversches Papier. 6 Zeilen auf Bl. 1 r°. Bl. 1 v° u. 37 r° leer. Auf Bl. 37 v° gegenläufig Notiz von Leibniz: „Gelieferte aber noch nicht bedungene bucher“.
Cc 2, Nr. 775 A tlw.

Octob. 1674.

De serierum summis et de quadraturis plagulae quindecim. In his multa notabilissima, inter alia hic de trochoide parabolica hic primum inventa plag. 10. 11. Et de curva ellips. et hyperbolae; plag. 13. et modus hic primum a me et meo marte inventus exhibendi valorem radicis quadraticae per seriem infinitam eadem plagula 13.

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20 plag. 10. 11.: N. 38_{12-13}. 21 plag. 13.: N. 38_{15}. 21 f. hic primum: Die Reihenentwicklung von Quadratwurzeln behandelt Leibniz auch in den wohl bereits kurz zuvor im Sommer 1674 enstandenen N. 32 (s. dort) und N. 33.

38_2. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS PRIMA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 2–3. 1 Bog. 2°. 4 S. Datum und Überschrift oben ergänzt.
Cc 2, Nr. 775 A tlw.

Octob. 1674.

Schediasma de serierum summis, et seriebus quadratricibus item curv. ellips. et hyperb. Et notabilissima extractio radicis quadraticae irrationalis per seriem infinitam plagula 13.

$$\square\ \text{hyp.}\ \underbrace{\frac{1}{1}-\frac{1}{2}}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+\frac{1}{9}-\frac{1}{10}+\frac{1}{11}-\frac{1}{12}+\frac{1}{13}-\frac{1}{14}$$

$$\frac{1}{2}\qquad\frac{1}{12}\qquad\frac{1}{30}\qquad\frac{1}{56}\qquad\frac{1}{90}\qquad\frac{1}{132}\qquad\frac{1}{182}$$

$$[\text{dupli}]\quad\frac{1}{1}\qquad\frac{1}{6}\qquad\frac{1}{15}\qquad\frac{1}{28}\qquad\frac{1}{45}\qquad\frac{1}{66}\qquad\frac{1}{91}$$

sunt triangulares per saltum. Nam series triangularium, est:

$$\frac{1}{1}\quad\frac{1}{3}\quad\frac{1}{6}\quad\frac{1}{10}\quad\frac{1}{15}\quad\frac{1}{21}\quad\frac{1}{28}\quad\frac{1}{36}\quad\frac{1}{45}\quad\frac{1}{55}\quad\frac{1}{66}\quad\frac{1}{78}\quad\frac{1}{91}.$$

$$\text{Ergo}\qquad\frac{1}{3}\qquad\frac{1}{10}\qquad\frac{1}{21}\qquad\frac{1}{36}\qquad\frac{1}{55}\qquad\frac{1}{78}$$

residua scilicet, pendent etiam a quadratura hyperbolae seu

$\frac{1}{1\frown 3}\qquad\frac{1}{2\frown 5}\qquad\frac{1}{3\frown 7}\qquad\frac{1}{4\frown 9}.$ Quae fiunt ex hac serie:

$1-\frac{1}{3}\ +\ \frac{1}{2}-\frac{1}{5}\ +\ \frac{1}{3}-\frac{1}{7}\ +\ \frac{1}{4}-\frac{1}{9},$ seu reducendo ad originem:

6 quadratricibus *(1)* aliae hic miscentur de quadraturis figurarum, de trochoide parabolica etc. plag. 10. 11. *(2)* item *L* 11 dimidii *L ändert Hrsg.* 15 hyperbolae *(1)*. Fiunt au *(2)* seu *L*

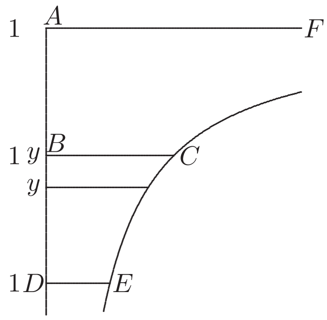
16 hac serie: Richtig wäre $1-\frac{2}{3}+\frac{1}{2}-\frac{2}{5}+\frac{1}{3}-\frac{2}{7}+\frac{1}{4}-\frac{2}{9}$ etc. Der Fehler und ähnliche, die Leibniz in den S. 387 Z. 1; S. 388 Z. 1–3 u. S. 388 Z. 9 f. unterlaufen, beeinträchtigen die weitere Überlegung nicht.

$\frac{b}{1} - \frac{b^3}{3} + \frac{b^2}{2} - \frac{b^5}{5} + \frac{b^3}{3} - \frac{b^7}{7} + \frac{b^4}{4} - \frac{b^9}{9}$, quorum formulam indagabimus ita:

$\frac{b}{1} + \frac{b^2}{2} + \frac{b^3}{3}$ etc. fit ex summa omnium $\frac{1}{1-y}$, et

$\frac{b^3}{3} + \frac{b^5}{5} + \frac{b^7}{7}$ etc. fit ex summa [omnium] $\frac{y^2}{1-y^2}$.

Cuius aliunde scio dimensionem ex hyperbolae quadratura pendere vel ideo duobus modis sumendo eius momentum semper reditur ad quadraturam hyperbolae nempe multiplicando per $1+y$, et per $1-y$. Unde inter ea duo momenta differentia est cylinder figurae.



[*Fig. 1*]

Differentia ergo inter duo spatia hyperbolica aequatur cylindro figurae $\frac{1}{1-y^2}$, nam momentum eius multiplicati per $1+y$, est $\frac{1}{1-y}$ momentum figurae $BDEC$ ex AF. Si multiplicetur per $1-y$, fit $\frac{1}{1+y}$ momentum figurae $BCDE$ ex DE, ponendo AB.

1 $-\frac{b^9}{9}$, *(1)* quae fiunt ex *(a)* hac div *(b)* divisione huius formulae : x $\sqcap$ $\frac{y}{a+}$ *(2)* quorum *L*

3 omnium *erg. Hrsg.* 3 f. $\frac{y^2}{1-y^2}$. *(1)* Cuius ut obiter dicam dimensio pendet ex quadratura hyperbolae, nam addendo $\frac{1}{1-y^2}$, idem est ac si adderetur 1, et *(a)* caetera, *(b)* $\frac{y^2}{1-y^2}$ duplicaretur *(aa)*. Iam *(bb)* quia $\frac{1}{1-y^2}$ est $\sqcap$ ipsi *(2)* Cuius *L*

$BD \sqcap 1$. Denique multiplicando per y fiet: $\dfrac{y^3}{1-y^2}$ momentum eiusdem figurae $BDCE$ ex BC. Unde: $y^3+y^5+y^7$ [etc.]

Sed ut ad rem redeamus[,] iungantur inter se hae duae figurae: $\dfrac{1}{1-y}$ et $\dfrac{y^2}{1-y^2}$ auferendo seu fiat $\dfrac{1}{1-y} - \dfrac{y^2}{1-y^2}$, sive reducendo ad communem denominatorem fiet: $\dfrac{1+y-y^2}{1-y^2} \sqcap x$. quae figura proinde pendet ex quadratura hyperbolae, reducendo fiet: $1+y-y^2 \sqcap x-y^2x$. et ut quaeratur valor ipsius y fiet:

$$y^2 \ \frac{+y}{x-1} + \frac{1}{4x^2-8x+4} \sqcap \boxed{\frac{x-1}{x-1}} \ 1+\frac{1}{4x^2-8x+4} \text{ sive } \sqcap \frac{4x^2-8x+5}{4x^2-8x+4}. \text{ Unde}$$

$y+\dfrac{[1]}{2x-2} \sqcap \dfrac{\sqrt{4x^2-8x+5}}{2x-2}$. sive $y \sqcap \left[\dfrac{\sqrt{4x^2-8x+5}-1}{2x-2}\right]$. quae pendet ex quad. hyp.

Etiam uti licet radicibus irrationalibus, aliquando, ut si consentiat radix numeratoris et denominatoris, e. g. $\dfrac{\sqrt{1-y^2}}{1-\sqrt{1-y^2}}$ et summa proveniet rationalibus irrationalibus mixta, v. g. $\sqrt{1-y^2}$ (circulus) $+1-y^2+\underbrace{1-y^2, \frown \sqrt{1-y^2}}+\underbrace{1-y^2, \frown 1-y^2}$ [etc.] atque ita rationales atque irrationales habebuntur per intervalla unius interstitii. Eodem modo radices cubicae tractari possunt.

Sed si radices numeratoris et nominatoris non consentiant, vel si radix non sit in utroque, utique patet seriem totam fieri irrationalem. Omnes autem rationales sunt semper paraboloeides.

Nemo usus est hactenus hyperboloeidibus in dividendo, cum tamen res non minus succedat: sed nec adhibitae sunt divisiones; in quibus ipsi producti habent denominatores,

2 etc. *erg. Hrsg.* 3 redeamus[,] *(1)* patet hanc figuram *(2)* iungantur *L* 8 y+ | $\dfrac{y}{2x-2}$ *ändert Hrsg.* | $\sqcap \dfrac{\sqrt{4x^2-8x+5}}{2x-2}$. *(1)* abiecto iam triangulo y, residua figura dabit y $\sqcap$ *(2)* sive y $\sqcap$ | $\dfrac{\sqrt{4x^2-8x+5}}{2x-1}$ *ändert Hrsg.* |. quae *L* 9 aliquando, ut *erg. L* 10 proveniet *(1)* rationalis, dividendo *(2)* rationalibus *L* 11 etc. *erg. Hrsg.* 12 interstitii *(1)*, quia radix quadratica, si *(2)*. Eodem *L* 15 patet *(1)* utrobique *(2)* seriem *L* 15 rationales *(1)* in casu praesenti *(2)* sunt *L*

nec adhibitae sunt irrationales. Imo non nisi unicum exemplum datum est; quod attulit Mercator. Methodus mea revocandi ad progressiones geometricas, commodior est altera Mercatoris per divisionem; quia, ita series qualescunque propositae etiam irregulares satis nec ordine procedentes, ad figuram convenientem, revocantur, qualis ista est: $\frac{b}{1}-\frac{b^3}{3}+\frac{b^2}{2}$ etc. Variae aliae coniunctiones institui possunt, ut ista:

$$\underbrace{\frac{1}{1}-\overbrace{\frac{1}{2}+\frac{1}{3}}-\frac{1}{4}}+\underbrace{\frac{1}{5}-\overbrace{\frac{1}{6}+\frac{1}{7}}-\frac{1}{8}}+\underbrace{\frac{1}{9}-\overbrace{\frac{1}{10}+\frac{1}{11}}-\frac{1}{12}}\text{ [etc.]}$$

$$\frac{3}{4}-\frac{1}{6}+\frac{3}{40}-\frac{1}{42}+\frac{3}{108}-\frac{1}{110}\text{ etc.}$$

Et ita semper novae erui possunt figurae. Sumtis seriebus fractionum quadraticarum unitate deminutarum:

$$\frac{1}{3}+\frac{1}{8}+\frac{1}{15}+\frac{1}{24}+\frac{1}{35}+\frac{1}{48}+\frac{1}{63}+\frac{1}{80}+\frac{1}{99}+\frac{1}{120}+\frac{1}{143}+\frac{1}{168}+\frac{1}{195}+\frac{1}{224}\text{ [etc.]}$$

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Omnium terminorum punctatorum habetur summa; item omnium terminorum □ notatorum; ac proinde et totius seriei; sed termini circulo notati pendent ex quad. circuli, termini ⌢ notati ex quad. hyperb.

Sed quid termini $\frac{1}{3}$ $\frac{1}{24}$ $\frac{1}{63}$ $\frac{1}{120}$ [etc.], sane sunt: $\frac{1}{1\frown 3}$ $\frac{1}{4\frown 6\sqcap 3\frown 8}$ $\frac{1}{7\frown 9}$ $\frac{1}{10\frown 12}$ [etc.]

6–387,2 etc. *erg. Hrsg. fünfmal* 17 notatorum; *(1)* item tot *(2)* ac *L* 17 seriei; *(1)* item *(2)* sed *L*

2 Mercator: *Logarithmotechnia*, 1668, prop. XIV–XVII S. 28–33 [Marg.]. 10–13 Vgl. u. a. *LSB* III, 1 N. 39 S. 166 Z. 3–9 u. *LQK* prop. XLII S. 88. 16 f. Omnium ... seriei: vgl. N. 15 S. 180 Z. 6 – S. 181 Z. 8.

$\frac{1}{1\frown 3}+\frac{1}{4\frown 6}\left[+\frac{1}{7\frown 9}\right]+\frac{1}{10\frown 12}$ etc. Unde $1-\frac{1}{3}+\frac{1}{4}-\frac{1}{6}\left[+\frac{1}{7}-\frac{1}{9}\right]+\frac{1}{10}-\frac{1}{12}$ [etc.]

Unde $\frac{b}{1}-\frac{b^3}{3}+\frac{b^4}{4}-\frac{b^6}{6}+\left[\frac{b^7}{7}-\frac{b^9}{9}\right]+\frac{b^{10}}{10}-\frac{b^{12}}{12}$ etc. Ex summa omnium:

$1\ -y^2\ +y^3\ -y^5\ [+y^6\ \ -y^8]\ +y^9\ -y^{11}$ etc. Iam summa omnium

$1\qquad +y^3\qquad [+y^6]\qquad +y^9$ etc. fit ex $\frac{1}{1-y^3}$. Et summa omnium:

$y^2\qquad y^5\qquad [y^8]\qquad y^{11}$ etc. fit ex $\frac{y^2}{1-y^2}$.

Iam $1-y^2\frown 1+y\sqcap 1\ \boxed{-y^2+y^2}\ -y^3$. Fiet ergo $\frac{1-y^2-y^4}{1-y^3}\sqcap x$. aequatio figurae, cuius series quadratrix est: $\frac{1}{3}\ \ \frac{1}{24}\ \ \frac{1}{63}$ etc.

Eodem modo investigabitur figura, cuius series quadratrix: $\frac{1}{8}\ \ \frac{1}{35}\ \ \frac{1}{80}$ [etc.]

Restat investiganda series $\frac{1}{15}\ \ \frac{1}{63}\ \ \frac{1}{143}\ \ \frac{1}{255}$ quae vadit etiam per saltus tertianos, cuius terminos in serie ita notavi: ∞. Denominatores ita resolventur:

$\frac{1}{3\frown 5}\qquad\frac{1}{7\frown 9}\qquad\frac{1}{11\frown 13}\qquad\frac{1}{15\frown 17}$ [etc.] Unde fit series praeparata:

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10 *Über* $\frac{1}{15}\ \ \frac{1}{63}\ \ \frac{1}{143}$:

$\qquad 4\quad 4$

$4\quad 8\quad 12$

1 $+\frac{1}{7\frown 9}$ *erg. Hrsg.* 1 $+\frac{1}{7}-\frac{1}{9}$ *erg. Hrsg.* 3 $+\frac{b^7}{7}-\frac{b^9}{9}$ *erg. Hrsg.* 4 $+y^6-y^8$ *erg. Hrsg.* 5 $+y^6$ *erg. Hrsg.* 6 y^8 *erg. Hrsg.* 9 etc. *erg. Hrsg.* 10 f. tertianos *(1)* et eius figura pendet ex quadratura circuli et hyperbolae iunctis *(2)*, cuius *L* 12 etc. *erg. Hrsg.*

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1 $1-\frac{1}{3}$: Richtig wäre $\frac{1}{2}-\frac{1}{6}+\frac{1}{8}-\frac{1}{12}+\frac{1}{14}-\frac{1}{18}+\frac{1}{20}-\frac{1}{24}$ etc.; s. o. Erl. zu S. 383 Z. 16. 6 $\frac{y^2}{1-y^2}$: Richtig wäre $\frac{y^2}{1-y^3}$. Leibniz rechnet mit dem Wert weiter und erhält als Folge weiterer Rechenfehler $\frac{1-y^2-y^4}{1-y^3}$ statt $\frac{1-y^2}{1-y^3}$.

$$\frac{1}{3}-\frac{1}{5}+\frac{1}{7}-\frac{1}{9}+\frac{1}{11}-\frac{1}{13}+\frac{1}{15}-\frac{1}{17} \text{ etc. Cuius origo est:}$$

$$\frac{b^3}{3}-\frac{b^5}{5}+\frac{b^7}{7}-\frac{b^9}{9}+\frac{b^{11}}{11}-\frac{b^{13}}{13}+\frac{b^{15}}{15}-\frac{b^{17}}{17} \text{ etc. facta ex summis omnium:}$$

$$y^2-y^4+y^6-y^8+y^{10}-y^{12}+y^{14}-y^{16} \text{ etc. } \sqcap \frac{y^2}{1+y^2}.$$

Quoniam autem series

$\frac{1}{3}$ $\frac{1}{15}$ $\frac{1}{35}$ $\frac{1}{63}$ $\frac{1}{99}$ $\frac{1}{143}$ etc. a me inventa est; et series

$\frac{1}{3}$ $\frac{1}{35}$ $\frac{1}{99}$ etc. pendet ex quadratura circuli, itaque series

$\frac{1}{15}$ $\frac{1}{63}$ $\frac{1}{143}$ [etc.] etiam pendebit ex quadratura circuli.

$\frac{1}{3}$ $\frac{1}{35}$ $\frac{1}{99}$ [etc.] resoluta dant:

$\frac{1}{1\frown 3}$ $\frac{1}{5\frown 7}$ $\frac{1}{9\frown 11}$ etc., cuius seriei origo est

$\frac{b}{1}-\frac{b^3}{3}\ +\ \frac{b^5}{5}-\frac{b^7}{7}\ +\ \frac{b^9}{9}-\frac{b^{11}}{11}$ etc. facta ex summis omnium:

$1-y^2\ +\ y^4-y^6\ +\ y^8-y^{10}$ etc.

Idem plane evenit, examinatis duabus alteris ad hyperbolam seriebus, $\propto$ et $\frown$; ut non sit opus immorari. Videamus quid fiat, ademtis:

3 f. $\frac{y^2}{1+y^2}$. *(1)* Eodem modo sumatur series, alia per saltus tertianos, quam ita notavi $\propto$. $\frac{1}{3}$ $\frac{1}{35}$ $\frac{1}{99}$ *(2)* Quoniam autem *(a)* constat seriem *(3)* series *L* 7–389,6 etc. *erg. Hrsg. fünfmal* 7 f. circuli. *(1)* Miror autem eandem ex una quam ex altera serie prodire figuram. Nam *(2)* $\frac{1}{3}$ *L* 9 etc., *(1)* unde *(2)* quorum *(3)* cuius *L* 13 fiat, *(1)* additis | vel ademtis *erg. L, streicht Hrsg.* | *(2)* ademtis *L*

1–3 $\frac{1}{3}-\frac{1}{5}$: Richtig wären $\frac{1}{6}-\frac{1}{10}+\frac{1}{14}-\frac{1}{18}$ etc., $\frac{b^3}{6}-\frac{b^5}{10}+\frac{b^7}{7}-\frac{b^9}{9}$ etc. und $\frac{y^2}{2}-\frac{y^4}{2}+\frac{y^6}{2}$ etc. $=\frac{y^2}{2(1+y^2)}$; s. o. Erl. zu S. 383 Z. 16. 5 inventa: N. 15 S. 180 Z. 6 – S. 181 Z. 8; vgl. auch *LSB* III, 1 N. 39 S. 166 Z. 6 u. *LQK* prop. XXXVI S. 82. 9 origo: Richtig wären $\frac{1}{2}\left(\frac{b}{1}-\frac{b^3}{3}+\frac{b^5}{5}\text{ etc.}\right)$ und $\frac{1}{2}\left(1-y^2+y^4\text{ etc.}\right)$; s. o. Erl. zu S. 383 Z. 16.

$$\frac{1}{3}\ \frac{1}{8},\quad \frac{1}{35}\ \frac{1}{48},\quad \frac{1}{99}\ \frac{1}{120},\quad \frac{1}{195}\ \frac{1}{224},\ \text{[etc.]}$$

$$\frac{5}{24}\quad \frac{13}{35 \frown 48}\quad \frac{21}{99 \frown 120}\quad \text{etc.; auferatur}$$

$$\frac{1}{24}\quad \frac{1}{35 \frown 48}\quad \frac{1}{99 \frown 120}\quad \text{[etc.] fiet:}$$

$$\frac{1}{3 \frown 2}\quad \frac{1}{35 \frown 4}\quad \frac{1}{99 \frown 6}\quad \text{etc., cuius seriei momentum ex vertice, pendet}$$

ex summa seriei quadratricis circuli. Unde patet seriem summatricem huius ipsius seriei $\frac{1}{3 \frown 2}\ \frac{1}{35 \frown 4}$ [etc.] ex quad. circuli summandam esse.

$$\frac{1}{1 \frown 2 \frown 3 \frown 4}\quad \frac{1}{5 \frown 6 \frown 7 \frown 8}\quad \frac{1}{9 \frown 10 \frown 11 \frown 12}$$

$$\frac{1}{24}\quad \frac{1}{35 \frown 48}\quad \frac{1}{99 \frown 120}\ .$$

Unde series quadratrix differentiarum inter circulum et hyperbolam, est:

$$\frac{5}{1,2,3,4}\quad \frac{13}{5,6,7,8}\quad \frac{21}{9,10,11,12}\quad \text{etc.}\quad \text{Unde si auferatur,}$$

$$\odot\quad \frac{3}{1,2,3,4}\quad \frac{3}{5,6,7,8}\quad \frac{3}{9,10,11,12}\quad \text{etc.}\quad \text{restabit:}$$

$$\frac{2}{1,2,3,4}\quad \frac{10}{5,6,7,8}\quad \frac{18}{9,10,11,12}\quad \text{etc.,}\quad \text{vel}$$

$$☽\quad \frac{2}{2 \frown 3 \frown 4}\quad \frac{2}{6 \frown 7 \frown 8}\quad \frac{2}{10 \frown 11 \frown 12}\quad \text{etc.}$$

Atqui habetur summa huius seriei:

$$\frac{1}{1,2,3}\quad \frac{1}{2,3,4}\quad \frac{1}{3,4,5}\quad \frac{1}{4,5,6}\quad \frac{1}{5,6,7}\quad \frac{1}{6,7,8}\ \text{etc.}$$

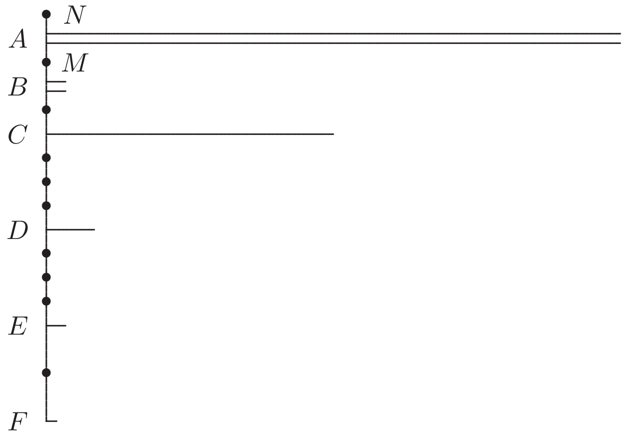
Hinc patet rursus iri per saltus tertianos ut series habeatur quaesita. Series autem huius quoque seriei habetur in

$$\frac{1}{1,2,3,4}\quad \frac{1}{2,3,4,5}\quad \frac{1}{3,4,5,6}\quad \frac{1}{4,5,6,7}\quad \frac{1}{5,6,7,8}\ \text{[etc.] unde rursus per saltus}$$

tertianos ut series habeatur quaesita.

5 patet *(1)* differentias seriei summatricis eius *(2)* seriem *L* 9 series *(1)* differentiarum inter quad. circuli *(2)* quadratrix *L* 16 ut ... quaesita *erg. L* 18 etc. *erg. Hrsg.*

Si series ⊙ triangulo-triangularium ducatur in distantias a vertice, 1, 5, 9. fient respondentes saltus tertiani pyramidalium; et si ☽ ducatur in distantias a vertice fient respondentes saltus triangularium.



[*Fig. 2*]

$$\begin{matrix} & C & D & E & F & & \\ ♀ & \frac{1}{1,2} & \frac{1}{3,4} & \frac{1}{5,6} & \frac{1}{7,8} & \frac{1}{9,10} & \frac{1}{11,12} \end{matrix}$$ etc. pendent a quadratura hyperbolae. Ducantur in distantias a vertice, A, nempe 2.4.6.8.10.12. (vel quod eodem redit, duplicando productum, 1.2.3.4) fiet: 1 $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{[7]}$ etc. momentum seriei ex A. Ducantur in distantias a B, fiet: $\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{6}$. $\frac{1}{8}$. $\frac{1}{10}$, seu [duplum] $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ etc.

Unde sequitur, quod et patet, differentiam duorum momentorum aequari seriei summae.

Esto series: $$\begin{matrix} & C & D & E & F \\ ☿ & \frac{2}{2,3,4} & \frac{2}{6,7,8} & \frac{2}{10,11,12} & \frac{2}{14,15,16} \end{matrix}$$ etc.

1 Si *(1)* figura *(2)* series ⊙ | triangulo-triangularium *erg.* | ducatur *L* 2 si *(1)* saltus pyrami *(2)* ☽ *L* 8 6 *L ändert Hrsg.* 9 seu *(1)* dimi *(2)* duplum *(3)* | dimidium *ändert Hrsg.* | $\frac{1}{1}$ *L*

Si ducantur in 2.4.6.8. etc. distantias a vertice A, fiet haec series:

$\frac{1}{2 \frown 3} \quad \frac{1}{6 \frown 7} \quad \frac{1}{10 \frown 11} \quad \frac{1}{14 \frown 15}$, etc. quae pendent ab his:

$\frac{1}{2} - \frac{1}{3} + \frac{1}{6} - \frac{1}{7} + \frac{1}{10} - \frac{1}{11} + \frac{1}{14} - \frac{1}{15}$ [etc.], cuius origo

$\frac{b^2}{2} - \frac{b^3}{3} + \frac{b^6}{6} - \frac{b^7}{7}$ etc. seu summa omnium, $y - y^2 + y^5 - y^6 + y^9 - y^{10}$ etc. seu $y + y^5 + y^9$ etc. demta $y^2 + y^6 + y^{10}$ etc. Quarum prior fit ex figura, cuius ordinata est: $\frac{y}{1-y^4}$, posterior ex figura, cuius ordinata est: $\frac{y^2}{1-y^4}$. Unde figura totalis $\frac{y-y^2}{1-y^4}$, quae dividi potest per $1-y$; fiet: $\frac{y}{y^3+y^2+y+1}$, cuius figurae series quadratrix est $\frac{1}{2} - \frac{1}{3} + \frac{1}{6} - \frac{1}{7}$ etc.

Hac arte comparari possunt eiusdem figurae series quadratrices diversae, prout scilicet instituitur divisio diverso modo. Deinde methodo geometrica cum figurae diversae ex se invicem pendeant hinc rursus multae series quadratrices diversae inter se conferri possunt.

Notabile est hoc loco theorema: $y^3 + y^2 + y + 1$. ducta in $y - 1$, dant $y^4 - 1$.

Unde patet momentum eiusmodi figurae cuius ordinata $y^3 + y^2 + y + 1$. absolute haberi posse ex axe aequilibrii transeunte per punctum quod sit infra punctum a quo incipit y, recta constante.

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7 *Nebenrechnung*:

$$\frac{1-y^4}{1-y} \sqcap \frac{\overset{-y^3+y^3}{\not{y}^{\not{4}}-1}}{\not{y}-1} \not{f}\; y^3, + \frac{\overset{-\not{y}^{\not{2}}+y^2}{\not{y}^{\not{3}}-1}}{\not{y}-\not{1}} \not{f}\; y^2, + \frac{y^2-1}{y-1} \not{f}\; y+1$$

1 fiet *(1)*: dimidium huius seriei *(2)* haec *L* 15 ex *(1)* puncto quod *(2)* axe *L* 15 punctum *(1)* verticis recta *(2)* quod *L*

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13 theorema: Sluse gibt eine allgemeine Formulierung und das Beispiel $(y^3 - x^3) : (y - x) = y^2 + yx + x^2$ im Nachtrag zu seiner Tangentenregel, die Leibniz S. 415 Z. 26 erwähnt; s. *Philosophical Transactions* VIII Nr. 95 vom 23. Juni/3. Juli 1673 S. 6059.

Hinc, si ducatur idem in y; fiet $y^4+y^3+y^2+y$, cuius figurae solidae summa, differet a cylindro figurae y^3+y^2+y+1. per cylindrum figurae y^4-1. sub recta constante quam appellamus 1. Cylinder autem figurae y^4-1. habetur absolute seu cubari potest. Ergo figura $y^4+y^3+y^2+y$. pendet a figura y^3+y^2+y+1. adeoque a figura y^3+y^2. Sed etsi haec iam tum habeantur haec tamen inquisitio ad alia quoque imo et ad ista de seriebus demonstranda, in numeris quoque utilis esse potest.

Porro si series ipsarum ☿ $C.D.E.F.$ ducatur in 1.3.5.7. fiet: $\frac{1}{3,4}\quad \frac{1}{7,8}\quad \frac{1}{11,12}$ [etc.] Quorum duorum momentorum $\frac{1}{6}\quad \frac{1}{42}\quad \frac{1}{[110]}$ etc. et $\frac{1}{12}\quad \frac{1}{56}\quad \frac{1}{132}$ [etc.] differentia aequatur cylindro seriei. At eorundem summa aequatur seriei $\frac{1}{2,3}\quad \frac{1}{3,4}\quad \frac{1}{6,7}\quad \frac{1}{7,8}\quad \frac{1}{10,11}\quad \frac{1}{11,12}$ [etc.]

Differentia autem inter summam et differentiam aequatur duplo minoris, et summa summae et differentiae aequatur duplo maioris.

Quod si interstitiola inter CD, DE, non tantum in 2, sed in 4 partes subdividantur, ut una partium v. g. sit BM vel NA, ponderando ab N, ex serie ☿ fiet aliud momentum nempe $\frac{1}{[1\frown 2]}\quad \frac{1}{3\frown 4}\quad \frac{1}{5\frown 6}$ [etc.] quod rursus a prioribus cylindro figurae certo modo sumto differt. Omnium serierum, quae in distantias ab aequilibrii axe ductae summari possunt, etiam series summatrices summari possunt.

$\frac{1}{1}\quad \frac{1}{4}\quad \frac{1}{9}\quad \frac{1}{16}$ etc. Habentur autem $\frac{1}{4-1}\quad \frac{1}{9-1}\quad \frac{1}{16-1}$. Unde summa harum: $\frac{4}{4-1}\quad \frac{9}{9-1}\quad \frac{16}{16-1}$. Habentur $\frac{4+1}{4-1} \sqcap \frac{2+1}{}$ [*bricht ab*]

11 f. *Nebenrechnungen*: $-a+b+b+a$
$a+b-b+a$

1 si *(1)* recta alia suma *(2)* ducatur *L* 1 solidae *erg. L* 2 a *(1)* summa *(2)* cylindro *L* 7–15 etc. *erg. Hrsg. viermal* 8 99 *L ändert Hrsg.* 9 seriei *(1)* supra expositae ♀ $\frac{1}{1,2}\quad \frac{1}{3,4}$ *(2)* $\frac{1}{2,3}$ *L* 13 tantum in *(1)* 4, sed in 8 *(2)* 2, *L* 15 $2\frown 3$ *L ändert Hrsg.* 16 differt. *(1)* Quandocunque *(2)* Omnium *L* 16 ductae *(1)* quadrari *(2)* summari *L*

38$_3$. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS SECUNDA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 4–5. 1 Bog. 2^{o}. 4 S. Datum u. Überschrift auf Bl. 4 r^{o} oben ergänzt.
Cc 2, Nr. 775 A tlw.

Octob. 1674.

De serierum summis et seriebus quadratricibus, pars 2da.

Urgenda sunt quae de seriebus summandis, et figuris ea ratione quadrandis, a me dicta sunt.

Habetur series ista:

$$(1)\qquad \frac{1}{4-1}\qquad \frac{1}{9-1}\qquad \frac{1}{16-1}\qquad \frac{1}{25-1}\qquad \text{[etc.]}$$

ut alibi a me demonstratum est.

Auferatur, a $$(2)\qquad \frac{4}{4-1}\qquad \frac{9}{9-1}\qquad \frac{16}{16-1}\qquad \frac{25}{25-1}\qquad \text{[etc.]}$$

restabit: $$1\qquad 1\qquad 1\qquad 1\qquad \text{[etc.]}$$

Hinc in terminis finitis, saltem facile ope seriei 1, habetur series 2.

Addantur series 1, et 2, fiet:

$$(3)\qquad \frac{4+1}{4-1}\qquad \frac{9+1}{9-1}\qquad \frac{16+1}{16-1}\qquad \frac{25+1}{25-1}\qquad \text{etc.}$$

Auferatur adhuc alia series

$$(4)\qquad \frac{4}{4-1}\qquad \frac{6}{9-1}\qquad \frac{8}{16-1}\qquad \frac{10}{25-1}\qquad \text{etc.}$$

fiet: $$(5)\qquad \frac{4-4+1}{4-1}\qquad \frac{9-6+1}{9-1}\qquad \frac{16-8+1}{16-1}\qquad \frac{25-10+1}{25-1}\qquad \text{[etc.]}$$

et dividi poterunt omnes per

$$(6)\qquad 2-1\qquad 3-1\qquad 4-1\qquad 5-1\qquad \text{[etc.]}$$

10–394,6 etc. *erg. Hrsg. siebenmal* 11 f. est. *(1)* Habetur *(2)* Auferatur *L* 12 f. $\frac{25}{25-1}$ | restabit: *streicht Hrsg.* | restabit *L* 16 f. etc. *(1)* sive *(2)* sive divisis omnibus per $2-1$ fiet *(3)* addatur *(4)* Auferatur *L* 18 f. etc. *(1)* Cuius seriei ut obiter dicam habetur summa, quia pendet a *(2)* fiet *L*

---

11 alibi: N. 15 S. 180 Z. 6 – S. 181 Z. 8; vgl. *LQK* prop. XLI S. 88.

fietque series ab unitatibus harmonicae differens

$$(7)\qquad \frac{2-1}{2+1}\qquad \frac{3-1}{3+1}\qquad \frac{4-1}{4+1}\qquad \frac{5-1}{5+1}\qquad \text{etc.}$$

Si addas seriem 4, fiet:

$$(8)\qquad \frac{4+4+1}{4-1}\qquad \frac{9+6+1}{9-1}\qquad \frac{16+8+1}{16-1}\qquad \frac{25+10+1}{25-1}\qquad [\text{etc.}]$$

et divisis omnibus per

$$(9)\qquad 2+1\qquad 3+1\qquad 4+1\qquad 5+1\qquad [\text{etc.}]$$

fiet series etiam ex harmonica pendens

$$\frac{2+1}{2-1}\qquad \frac{3+1}{3-1}\qquad \frac{4+1}{4-1}\qquad \frac{5+1}{5-1}\qquad \text{etc.}$$

Momentum seriei $\frac{y^2+1}{y^2-1}$ seu series ducta in $y-1$ dat $\frac{y^2+1}{y+1}$, quae pendet ex serie harmonica, nam $\frac{y^2+1+2y}{y+1} \sqcap y+1$. Unde patet si seriei $\frac{y^2+1}{y+1}$ addatur series ex harmonica pendens: $\frac{2y}{y+1}$ fieri seriem finite summabilem.

Momentum eiusdem seriei seu $\frac{y^2+1}{y^2-1}$ ductum in $y+1$, est $\frac{y^2+1}{y-1}$, differentia autem inter: $\frac{y^2+1}{y+1}$ et $\frac{y^2+1}{y-1}$ erit $\frac{y^2+1}{y-1} - \frac{y^2+1}{y+1}$, seu $\frac{y^3+y^2+y+1-y^3+y^2-y+1}{y^2-1}$ seu $\frac{2y^2+2}{y^2-1}$.

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13 *Nebenrechnung*:

$$\begin{array}{r} y^2+1 \\ y\ \ -1 \\ \hline y^3+y-y^2-1 \end{array}$$

1 series *(1)* harmonica *(2)* ab *L* 8 f. etc. *(1)* Si a serie 4. dimidiata auferas seriem 1. duplicatam, fiet series harmonica pura, seu naturalis. Nam $\frac{2}{4-1} - \frac{1}{-}$ *(2)* Momentum *L* 9 seu series *erg. L*
12 y + 1, *(1)* dat y *(2)* est *L*

Sed hinc video ex ipso semper in his calculo patere, quod differentia momentorum aequetur [cylindro] seriei, nec proinde quicquam inde magnopere duci posse.

$\frac{y^2-1}{y^2-1} \sqcap 1$. Habetur $\frac{1}{y^2-1}$ ergo et habetur $\frac{y^2}{y^2-1}$. Ad $\frac{y^2}{y^2-1}$ cognitam addatur $\frac{y}{y^2-1}$, fiet: $\frac{y^2+y}{y^2-1} \sqcap \frac{y}{y[-]1}$. Ergo si inveniatur $\frac{y}{y^2-1}$, invenietur et series harmonica. Hinc patet $\frac{y}{y^2-1}$ pendere ex serie harmonica.

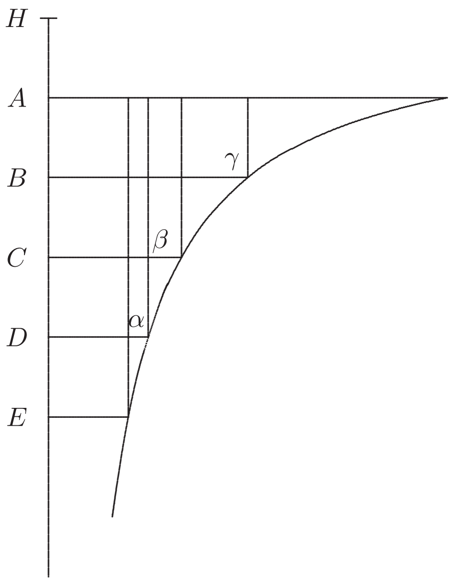
$\frac{1}{z^2+2z}$, est series quadratorum unitate diminutorum ponendo $y \sqcap z+1$, cuius seriei manifestum est haberi summam, cum sit triangularis.

Iam pro $\frac{y}{y^2-1}$ faciamus $\frac{z+1}{z^2+2z}$. Patet in duas dissolvi series, $\frac{1}{z+2}$, et $\frac{1}{z^2+2z}$, ex quibus posterioris habetur summa, prior est harmonica.

$\frac{1}{a+z}\quad \frac{1}{a+2z}\quad \frac{1}{a+3z}\quad \frac{1}{a+4z}\quad \frac{1}{a+5z}$.

$\frac{1}{z}$ [−] $\frac{1}{z+1} \sqcap \frac{z+1-z}{z^2+z} \sqcap \frac{1}{z^2+z}$. Ducantur differentiae in z fiet: [*bricht ab*]

1 in his *erg. L* 2 cylindro *erg. Hrsg.* 2 f. posse. *(1)* Sed hoc notabile est, quod diff *(2)* $\frac{y^2-1}{y^2-1}$ *L* 3 $\frac{y^2}{y^2-1}$. *(1)* Si *(a)* a serie $\frac{y^2+2y+1}{y+1}$, auferatur *(b)* seriei $\frac{y^2+2y+1}{y+1}$, addatur $\frac{-4y}{y+1}$ fiet: *(2)* Ad $\frac{y^2}{y-1}$ | cognitam *erg.* | addatur *L* 4 + *L ändert Hrsg.* 4 f. harmonica. *(1)* Sed $\frac{y}{y+1 \frown y-1} \sqcap y^2+y$ *(2)* Quaerendae sunt duae quantitates b et c, ita ut b, $\frown$ y + 1, −, c $\frown$ y − 1 $\sqcap$ y *(3)* Hinc *L* 9 f. harmonica. *(1)* $\frac{1}{z+1}\ \frac{1}{z+2}\ \frac{1}{z+3}\ \frac{1}{z+4}$ *(2)* $\frac{1}{a+z}$ *L* 11 − *erg. Hrsg.* 11 differentiae *erg. L* 11 z | vel in z + 1 *gestr.* | fiet: *L*



[*Fig. 1*]

Notabile est in serie harmonica, quod altitudo HC in diff. $\beta \sqcap C\beta$ in β seu in 1. Ergo $HC \sqcap \dfrac{C\beta \frown 1}{\beta}$.

Datur momentum seriei harmonicae ex vertice; quod si iam et quadrata terminorum huius seriei dentur $\frac{1}{1}\quad\frac{1}{4}\quad\frac{1}{9}\quad\frac{1}{16}\quad\frac{1}{25}$ dabitur centrum gravitatis cuiusdam portionis.

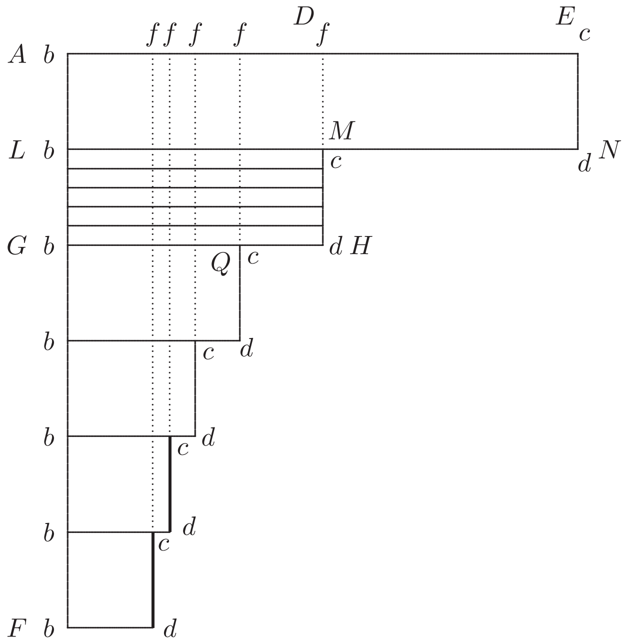
Semiquadrata abscissarum ducta in differentias aequantur figurae momento ex vertice. Momentum autem seriei harmonicae ex A, est rectangulum: semiquadrata abscissa-

2 altitudo *erg. L* 2 diff. *erg. L* 3 f. $\dfrac{C\beta \frown 1}{\beta}$. *(1)* Datur autem summa omnium HC, ergo et omnium $\frac{1}{z}$ divisorum per $\dfrac{1}{z^2+z}$ *(2)* Datur *L* 4 vertice; *(1)* datur et *(2)* quod *L*

2 diff. β: Leibniz bezeichnet mit diff. β die Ordinatendifferenz, anschließend mit $\beta = 1$ die Abszissendifferenz.

rum sunt $\frac{y^2}{2}$ fiet: $\frac{y^2}{2} \frown \frac{1}{y^2+y}$ sive $\frac{y}{2y+[2]}$. Quorum haberi potest summa. Eodem modo abscissae ductae in ordinatas [et differentias] aequantur ordinatarum semiquadratis.

Sed ne erremus in duabus istis regulis sane maximi momenti ad sequentia futuris.



[*Fig. 2*]

In figura hic exposita patet

(1) momentum seriei ex vertice AE, aequari summae ordinatarum bc, in unitatem cd, et distantiam a vertice AE, seu abscissam ductarum.

(2) Patet etiam momentum seriei ex axe AF aequari summae ipsarum bc, semiquadratorum in unitatem cd ductorum. Iam ex punctis c, ducantur in axem AE, aliae rectae cf punctis designatae. Unde patet

4 *Unter Fig. 2*: NB. In hac figura utile est fingi $AE \sqcap cd$, seu unitati.

1 1 *L ändert Hrsg.* 2 in *(1)* ordinatarum differentias aequantur *(2)* unitatem *(3)* ordinatas | et differentias *erg. Hrsg.* | aequantur *L* 8 ex *(1)* vertice *(2)* axe *L* 8 summae *erg. L*

(3^{tio}) momentum seriei ex vertice, aequari summae solidorum quae fiunt ex semiquadratis abscissarum $cf \sqcap bA$ in differentias ordinatarum cd ductis. Patet et

(4^{to}) momentum seriei ex axe, aequari summae solidorum, quae fiunt ex ordinatis, abscissis, et differentiis ordinatarum.

Sed video adhuc aliquid subesse difficultatis. Sume rectangulum $LGHM$. Eius momentum ex ipsa LM, est eius dimidium prisma, seu $LGHM$, ductum in dimidiam unitatem, seu posita unitate $\sqcap b$. et $GH \sqcap x$. erit $\frac{xb^2}{2}$. Ergo momentum eius ex AE, erit Ab abscissa, in ipsum rectangulum ducta, ipsa $\frac{xb^2}{2}$ [minuta]: seu $yxb - \frac{xb^2}{2}$, seu $y - \frac{b}{2}$ in xb. Ergo

(1) momentum seriei ex vertice componitur solidis ex rectangulo unitatis seu differentiae abscissarum et ordinatae ducto [in] abscissam dimidia unitate seu dimidia abscissarum differentia minutam.

Et pari iure

(2) momentum seriei ex axe, componitur solidis ex rectangulo differentiae ordinatarum et abscissae ducto in ordinatam dimidia ordinatarum differentia minutam. Deinde

(3) momentum seriei ex axe componitur solidis ex semiquadratis ordinatarum, in unitatem seu differentiam abscissarum. Et pari iure

(4) momentum seriei ex vertice, componitur solidis ex semiquadratis abscissarum in differentiam ordinatarum. Ergo

(5) summa solidorum quae fiunt ex rectangulis sub unitate seu differentia abscissarum et ordinata; in abscissam dimidia unitate minutam ductis aequatur summae solidorum quae fiunt ex semiquadratis abscissarum in differentias ordinatarum.

1 summae *erg. L* 3 axe, *(1)* fieri ex *(a)* du *(b)* solidis *(2)* aequari *L* 8 $\frac{\mathrm{xb}^2}{2}$ | aucta *ändert Hrsg.* |: *(1)* seu $\mathrm{yxb} + \frac{\mathrm{xb}^2}{2}$, seu $\mathrm{y} + \frac{\mathrm{b}}{2}$ *(2)* seu *L* 10 vertice | *(1)* demta sequenti ordinata *(2)* demto rectangulo sequentis ordinatae in praesentem abscissam *erg. u. gestr.* | *(a)* est *(aa)* rect *(bb)* solidum ex rectangulis *(b)* componitur *L* 10 f. seu differentiae abscissarum *erg. L* 11 ordinatae *(1)* ducta in dimidium *(2)* ducto | in *erg. Hrsg.* | abscissam *L* 12 differentia *(1)* auctam *(2)* minutam. 14 axe, *(1)* est solidum ex rectangulis et differentiae ordinatarum et abscissae ductum *(2)* componitur *L* 15 differentia *(1)* auctam *(2)* minutam *L* 16 axe *(1)* est solidum *(2)* componitur *L* 18 vertice, *(1)* est solidum ex semiquadratis ordinata *(2)* componitur *L* 20 (5) *(1)* solidum ex rectangulo unitatis seu differentiae *(2)* summa *L* 21 unitate *(1)* auctam *(2)* minutam *L*

(6) Summa solidorum quae fiunt ex rectangulis sub differentia ordinatarum et abscissa, in ordinatam dimidia ordinatarum differentia minutam; aequatur summae solidorum quae fiunt ex semiquadratis ordinatarum in unitates seu differentias abscissarum.

(7) Summa rectangulorum sub abscissis et differentiis ordinatarum, aequatur summae rectangulorum sub ordinatis et unitatibus vel differentiis abscissarum sive aequatur summae seriei.

Series numerorum figuris geometriae in eo sunt intractabiliores, quod in numeris non licet mutare abscissam in ordinatam et vicissim; nec alium pro alio axem sumere, saltem regulariter.

Esto iam e. g. $AG \sqcap y$, et $GH \sqcap \frac{1}{y}$ et ita de caeteris. Differentia ordinatarum erit $\frac{1}{y} - \frac{1}{y+1} \sqcap \frac{1}{y^2+y}$. Momentum seriei ex axe, erit summa omnium $\frac{1}{2y^2}$; quae aequabitur summae omnium: $y, \frown \frac{1}{y^2+y}, \frown \frac{1}{y} - \frac{1}{2y^2+2y}$, fiet: $\frac{1}{y+1} \frown \frac{1}{y} - \frac{1}{2y^2+2y} \sqcap \frac{2y\boxed{+2-1}\ +1}{2y^2+2y} \frown \frac{1}{y+1} \sqcap \frac{y}{2y^2+2y} \frown \frac{1}{y+1} + \frac{1}{2y^2+2y} \sqcap \frac{1}{2y^2+4y+2} + \frac{1}{2y^2+2y}$. Quorum summa aequari debebit summae omnium $\frac{1}{2y^2}$.

Sed iam video aliquid ad nostras regulas addendum esse; nimirum differentiarum ordinatarum summae.

Imo video duas priores, et per consequens et 5^{tam} 6^{tamque} plane vacillare. Videamus tamen: Manifestum est momentum rectanguli LAE esse ipsum rectangulum ductum in abscissam, 1, dimidiatam. Et momentum rectanguli GLM ex AE, esse rectangulum

2 differentia *(1)* auctam *(2)* minutam *L* 4 (7) *(1)* Abscissae in differentias ordinatarum, *(2)* Summa *L* 5–7 aequatur *(1)* portioni fig *(2)* summae seriei. *(a)* Nota: in seriebus *(b)* Series *L* 7 in numeris *erg. L* 8 sumere, *(1)* nisi qui sit dato parallelus *(2)* saltem *L* 10 f. caeteris. | Differentia *(1)* abscissarum *(2)* ordinaturum, erit $\frac{1}{\mathrm{y}} - \frac{1}{\mathrm{y}+1} \sqcap \frac{1}{\mathrm{y}^2+\mathrm{y}}$. *erg.* | Momentum *(a)* figurae *(b)* seriei *L* 12 summae *(1)* horum solidorum, x $\frown$ *(2)* omnium: *L* 15 $\frac{1}{2\mathrm{y}^2}$. *(1)* Proba quaedam veritatis ita statim haberi potest, si duo prima absolute aequentur. Quod cum non fiat, patet errorem esse in regulis, qui sic corrigetur. Si primum momentum $\frac{1}{\mathrm{y}^2}$, aequetur *(2)* Sed *L*

17 Imo video: vgl. aber unten S. 400 Z. 13.

GLM, ductum in rectam AL, abscissam suam unitate minutam, et praeterea ipsum rectangulum in dimidiam unitatem ductum. Appellando ergo abscissam y, ordinatam x, fiet: $xb, \frown y - b,, + \frac{b^2x}{2} \sqcap xyb - \frac{b^2x}{2}$. Ita momentum figurae $ALGHME$ est $\frac{1}{2} + 1 - \frac{1}{4} \sqcap \frac{5}{4}$ ex aequilibrii axe scilicet AE.

Item momentum, rectanguli LE, ex AE, librati, fit etiam ex d i f f e r e n t i a o r d i n a t a r u m e hoc loco $\frac{1}{2}$, [ducta] in semiquadratum abscissae $AL \sqcap y, \frac{y^2}{2}$, fit $\frac{y^2}{4}$ addito praeterea eodem semiquadrato AL in minimam ordinatarum LM ducto.

Si spatium propositum sit, brevius: abscissae semiquadratum ducitur in differentias ordinatarum, et si nulla sequitur, in minimam ordinatam, quia enim nulla sequitur[,] ipsa differentia est inter se ipsam et nihil, quod sequitur. Hinc $ALGHME$ momentum fit ex $MN \sqcap \frac{1}{2}$ in $\frac{b^2}{2} \sqcap \frac{1}{2}$ seu $\frac{1}{4}$; et praeterea $GH \sqcap \frac{1}{2}$, ducta in 2, semiquadratum ipsius AG quod est $\frac{1}{2}$, unde 2 in $\frac{1}{2}$ dat 1[,] fit ergo $\frac{1}{4} + 1$. seu $\frac{5}{4}$. ut ante.

Regulae ergo 6 superiores verae sunt absolute, nec opus habent correctione modo illud intelligatur; ultimam ex differentiis ordinatarum, esse ipsam ordinatam ultimam, minimam, si a vertice decrescitur, aut maximam si crescitur. Eodem modo inter differentias abscissarum, u l t r a u n i t a t e s esse abscissam primam, quae scilicet revera est u n i t a s , ut Ed, quando ad axem aequilibrii assurgit figura. Itaque in semiquadratis ordinatarum haec nihil turbant.

Illud tantum notandum est, quod dixi differentiis ordinatarum connumerandam esse ultimam ordinatam, id verum esse simpliciter non tantum de earum semiquadratis.

Nunc calculum resumamus: Momentum figurae nostrae ex axe, est $\frac{1}{2y^2}$, summa scilicet semiquadratorum ordinatarum. Quod aequatur summae horum:

2 ergo *(1)* ordinatam y, abscissam *(2)* abscissam *L* 4 ex *(1)* axe rect *(2)* aequilibrii *L* 5 momentum, *(1)* fit ex semiquadrato abscissae ducto *(2)* rectanguli *L* 5 f. d i f f e r e n t i a *(1)* a b s c i s s a r u m *(2)* o r d i n a t a r u m *L* 6 ductam *L ändert Hrsg.* 14 ultimam, *erg. L* 15 modo *(1)* ultimam ex differentiis *(2)* inter *L* 16 abscissam *(1)* minimam uni *(2)* primam, *L* 17 quando *(1)* ad verticem sci *(2)* ad *L* 18 f. turbant. *(1)* His ita positis resumendus est calculus, nempe in casu nostro, momentum figurae ex axe, est summa omnium $\frac{1}{2y^2}$. seu semiquadratorum in unitatem: Idem vero momentum etiam est summa *(2)* Illud *L* 20 ordinatam *erg. L*

$\boxed{\frac{1}{y} - \frac{1}{y+1}}\ \frac{1}{y^2+y}, \frown y, , \frown \frac{1}{y} - \frac{1}{2y^2+2y}$, sive $\frac{1}{y^2+y}, \frown \frac{1}{1} - \frac{1}{2y+2}$. Iam $1 - \frac{1}{2y+2} \sqcap \frac{2y+2-1}{2y+2} \sqcap \frac{2y+1}{2y+2}$, fiet, $\frac{1}{y^2+y}, , \frown \frac{2y+1}{2y+2}$, sive $\boxed{\frac{1}{y^2+y} \frown \frac{y}{2y+2}}\ \frac{1}{2y^2+4y+2}, + \frac{1}{y^2+y} \frown \frac{y+1}{2y+2}$, sive denique $\frac{1}{y+1,\square,\frown 2} + \frac{1}{y^2+y,\frown 2}$.

Quod si ergo summae omnium $\frac{1}{y+1,\square \frown 2} + \frac{1}{y^2+y,\frown 2}$ addatur novissimae ordinatae $\frac{1}{y}$ semiquadratum $\frac{1}{2y^2}$ in novissimam abscissam ductum, seu $\frac{1}{2y}$ habebitur quantitas aequalis summae omnium $\frac{1}{2y^2}$. Quod si verum est dabitur modus inveniendi summam fractionum quadraticarum, etiam infinitarum.

Sumamus: $\underbrace{\frac{1}{1}\ \frac{1}{4}\ \frac{1}{9}\ \frac{1}{16}}_{\frac{820}{576}}$, quorum dimidium $\frac{410}{576}$, sed relinquamus totum.

8–402,3 *Nebenrechnungen*:

$$\begin{array}{rrrr}
16 & 16 & 16 & 9 \\
9 & 9 & 4 & 4 \\
\hline
144 & 144 & 64 & 36 \\
4 & 1 & 1 & 1 \\
\hline
576 & 144 & 64 & 36
\end{array}
\qquad
\begin{array}{r}
576 \\ 144 \\ 64 \\ 36 \\ \hline 820
\end{array}
\qquad
\begin{array}{r}
144 \\ 64 \\ 36 \\ \hline 244
\end{array}$$

$$\frac{820}{576} \sqcap 1 + \frac{244}{576}$$

$$\begin{array}{rcr}
\not{2} & & \\
\not{6}100 & \not{\vdash} & 1525 \\
4)\ \not{4}\not{4}4 & & 144 \\
\hline
 & & 1669
\end{array}
\qquad
\begin{array}{rcr}
\not{2} & & \\
\not{1}\not{4}\not{4}00 & \not{\vdash} & 3600 \\
\not{4}\not{4} & &
\end{array}$$

5 semiquadratum *(1)* in *(a)* diff *(b)* abscissam novi *(2)* $\frac{1}{2y^2}$ *L* 5 habebitur *(1)* series *(2)* sum *(3)* quantitas *L*

---

6 dabitur: Die Folgerung ist nicht richtig.

Iam et summam ineamus omnium $\dfrac{1}{[y]+1,\square}$, seu $\dfrac{1}{4}\ \dfrac{1}{9}\ \dfrac{1}{16}\ \dfrac{1}{25}$.

$\dfrac{244}{576}\,\overset{4}{\smile}\!\!\lceil\ \dfrac{61}{144}$, addatur $\dfrac{1}{25}$, fiet: $\dfrac{25\frown 61+144}{144\frown 25} \sqcap \dfrac{1669}{3600}$ addantur $\dfrac{1}{y^2+y}$ seu $\dfrac{1}{2}+\dfrac{1}{6}+\dfrac{1}{12}$ $+\dfrac{1}{20}$ id est differentia inter 1 et $\dfrac{1}{5} \sqcap \dfrac{4}{5}$.

---

2 *Nebenrechnungen zur Stufe (1) der Lesart, gestrichen*:

2	5	2	2	2	850
5	10	10	5	5	340
10	50	20	10	10	170
10	17	17	17	100	100
100	850	340	170		1460
17					
1700					

2 f. *Nebenbetrachtung, gestrichen*: $\dfrac{1}{y}-\dfrac{1}{y+1} \sqcap \dfrac{y+1-y}{y^2+y}$

$\frac{1}{1}$		$\frac{1}{2}$		$\frac{1}{3}$		$\frac{1}{4}$		$\frac{1}{5}$
	$\frac{1}{2}$		$\frac{1}{6}$		$\frac{1}{12}$		$\frac{1}{20}$	
	1		3		6		10	15
	1		4		10		20	35

1 y^2 *L ändert Hrsg.* 2 $\frac{1}{2}+$ *(1)* $\frac{1}{5}+\frac{1}{10}+\frac{1}{17}$. Ergo $\frac{1669}{3600}+\frac{1460}{1700} \sqcap \frac{820}{}$ *(2)* Hinc addatur novissimae $\frac{1}{4}$ quadratum $\frac{1}{16}$ in novissimam abscissam *(3)* $\frac{1}{6}$ *L*

1+3 seu $\frac{1}{4}\ \frac{1}{9}\ \frac{1}{16}\ \frac{1}{25}$: Leibniz summiert mit $\frac{1}{25}$ und im folgenden mit $\frac{1}{20}$ jeweils einen Term zuviel auf und verfehlt dadurch die Verifikation; er erkennt die Fehlerhaftigkeit der Rechnung, klärt aber die Ursache nicht.

Denique addatur novissimae ordinatae $\frac{1}{4}$, quadratum $\frac{1}{16}$ in novissimam abscissam 4, ductum, fiet $\frac{1}{4} \sqcap \frac{900}{3600}$, fiet $\frac{2569}{3600}$.

Error in calculo haud dubie. Sed nihil ista inquisitio ad rem pertinet, quia nihil contribuit ad summas quadratorum.

$$\frac{1}{y+1} \sqcap \frac{1}{y} - \frac{1}{y^2} + \frac{1}{y^3} - \frac{1}{y^4} + \frac{\frac{1}{y^4}}{y+1}.$$

Methodus ista Mercatoris ad summas numerorum non aeque facile ac ad summas linearum accommodari potest.

$$\frac{y}{1+y} \sqcap + y - \left[y^2 + y^3 - y^4 + \frac{y^5}{1+y}\right].$$

Si subtractiones et additiones alternae continuentur in infinitum, et y sit numerus, patet duarum infinitarum serierum differentiae dari posse numerum fractum aequalem si y sit fractio ut $\frac{\frac{1}{2}}{1+\frac{1}{2}} \sqcap \frac{1}{2}$ [− etc.], idem intelligi potest, et series quaelibet tam affirmativa quam negativa erit decrescens.

8 $y^3 + y^4 - y^5 + \frac{y^6}{1+y}$ *L ändert Hrsg.* 11 − etc. *erg. Hrsg.*

6 Mercatoris: *Logarithmotechnia*, 1668, prop. XIV – XVII S. 28–33 [Marg.].

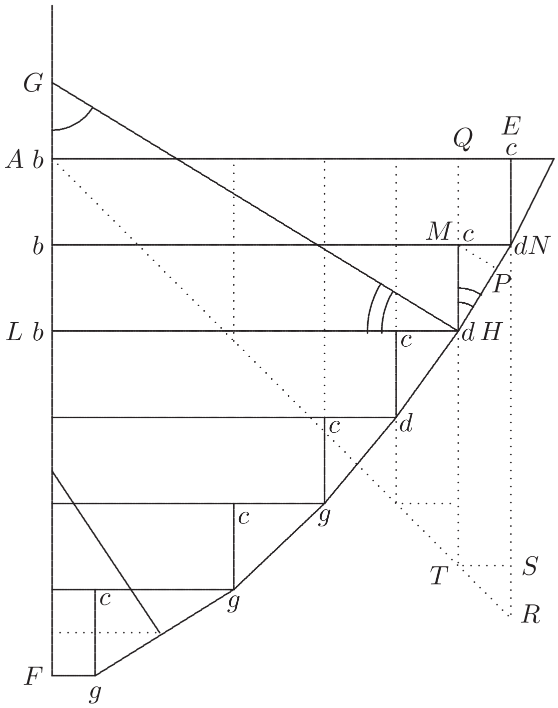
38_4. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS TERTIA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 6–7. 1 Bog. 2°. 3 S. — Auf Bl. 7 v° verworfene Nebenbetrachtung zu N. 38_5.
Cc 2, Nr. 775 A tlw.

Octob. 1674

Pars 3^{tia} schediasmatis de summis serierum.

Videndum est an quaedam transformationes geometricae applicari possint seriebus numerorum, praeter centrobarycas iam explicatas.



[*Fig. 1*]

Exposita serie instar figurae iungantur puncta extremitatum, ductis quasi quibusdam tangentibus seu polygoni lateribus. Ordinatae *bc*[,] differentiae earum *cd*, latera polygoni *dd*. Ducantur perpendiculares ut GH, dum axi AF producto si opus occurrat in G. Cum GHN sit rectus, erit triangulum GLH simile ∇^{lo} NMH, nam recta MP ipsi GH parallela. Ergo angulus M aequalis angulo G, ergo et angulus MHN aequalis angulo LHG.

Hinc $\dfrac{LH}{GL} \sqcap \dfrac{MH}{MN}$. Ergo $LH \frown MN \sqcap GL \frown MH$. Sed LH perpetuo differentiis ordinatarum ut MN, in axe ut EQ sumtis applicatae dant triangulum rectangulum semiquadratum quale est AER. Unde summa omnium $LH \frown MN$ aequatur semiquadrato maximae LH, modo inde summa omnium semiquadratillorum ipsarum $TS \sqcap SR \sqcap MN$ auferatur. Quare semper haberi poterit summa omnium GL, modo et quadrata differentiarum seriei datae haberi possint.

Sumamus exemplum in quo quadrata differentiarum habentur. Sit figura proposita trilineum concavum parabolicum arithmeticum, quod procedit, ut quadrata, differentiae procedunt ut triangula, seu ut numeri impares, horum ergo quadratorum haberi poterit summa. Ordinatae v. g. y^2, differentiae duarum ordinatarum $\boxed{y^2}\ +2y+1\ \boxed{-y^2}$ unde series differentiarum $2y+1$. Iam $GL \sqcap \dfrac{LH \frown MN}{1} \sqcap 2y+1, \frown y^2$. Horum ergo habetur summa.

In serie harmonica differentiae $\dfrac{1}{y^2+y}$, ducantur in $\dfrac{1}{y}$ fiet: $\dfrac{1}{y^3+y^2}$, cuius quantitatis summa pendet a quadratis triangularium. Unde patet, si hinc summae ipsarum $GL \frown MH$, illinc seriei ipsarum $LH \frown MN$, addantur semiquadratilla differentiarum, fieri ex his triangulum, ex illis seriem absolute triangulo aequalem. Ideoque regulam generalem hanc

5 GL ⌒ MH. *(1)* Hinc summa *(2)* Sed *L* 7 omnium *(1)* GL ⌒ MH, *(2)* LH ⌒ MN aequatur *(a)* rectangu *(b)* triangu *(c)* semiquadrato *L* 18 triangularium. *(1)* Et summa *(2)* Unde *L* 20 illis *(1)* quantitatem *(2)* seriem *L*

---

20 regulam generalem: Der folgende Satz entspricht der Formel $\sum_{i=0}^{n-1} x_i(x_{i+1}-x_i) + \sum_{i=0}^{n-1} \frac{1}{2}(x_{i+1}-x_i)^2 = \frac{1}{2}(x_n^2 - x_0^2)$. Leibniz setzt stillschweigend den Anfangswert $x_0 = 0$ voraus. In den anschließenden Beispielrechnungen übersieht er, daß bei der Differenzenbildung auf Vorzeichen zu achten ist; weitere Flüchtigkeitsfehler beeinträchtigen die Überlegungen bis S. 406 Z. 14. In N. 38_7 S. 438 Z. 9 nimmt Leibniz die Betrachtung wieder auf.

condere possum. I n q u a l i b e t s e r i e s i d i f f e r e n t i a e t e r m i n o r u m i n i p s o s t e r m i n o s r e s p o n d e n t e s d u c a n t u r f a c t i s a d d a n t u r d i f f e r e n t i a r u m s e m i q u a d r a t a ; s u m m a o m n i u m e r i t a e q u a l i s t e r m i n i m a x i m i s e m i q u a d r a t o.

Ideo haec series $\dfrac{1}{y^3+y^2}+\dfrac{1}{y^4+2y^3+y^2}\sqcap\dfrac{1}{2b^2}$ ponendo b esse maximam y seu $\dfrac{y^2+y+y}{y^5+2y^4+y^3}\sqcap\dfrac{y+2}{y^4+2y^3+y^2}$ cuius seriei habetur summa. Tractabiliores hoc casu series erunt si pro $\dfrac{1}{y^2+y}$ sumatur $\dfrac{1}{y^2-y}$.

Ex hac regula et haec sequitur pari iure:

Si unitates in abscissas respondentes ducantur factis addantur unitatum semiquadrata; summa omnium erit aequalis abscissae maximae semiquadrato.

Sed hoc dudum notum per se.

Si faciamus $\dfrac{1}{y-1}-\dfrac{1}{y}$, fiet: $\dfrac{\boxed{y-y}\;+1}{y^2-y}$, ducatur in $\dfrac{1}{y-1}$ fiet: $\dfrac{1}{y^2-2y+1,\frown y}$, addatur $\dfrac{1}{y^2-2y+1,\frown y^2}$ fiet: $\dfrac{y+1}{y^2-2y+1,\frown y^2}$. Huius vel superioris seriei, quarum datur summa, figuras geometricas quaeramus: $\dfrac{2}{1\boxed{-2+2},\frown 1}+\dfrac{3}{\boxed{4-4}+2,\frown 2}+\dfrac{4}{\boxed{9-6}\;+2,\frown 3}$.
$\qquad\qquad\qquad\qquad\qquad\qquad 3$

Notandum hic ut obiter dicam, satis difficile fore propositam numerorum seriem: v. g. hoc loco $\dfrac{2}{1}+\dfrac{3}{4}+\dfrac{4}{15}$ etc. revocare ad regulam seu aequationem. Esset haec quasi portio quaedam artis faciendi hypotheses sive artis decyphrandi.

Investigemus paulo accuratius quot modis fieri possit, ut seriei cuiusdam differentiae dent alias series.

Sit: $\dfrac{y\textcircled{v}}{\boxed{\textcircled{v}}\,y}-\dfrac{y+1\textcircled{v}}{y+1\boxed{\textcircled{v}}}$, unde $\dfrac{y\textcircled{v},\frown y+1\boxed{\textcircled{v}},,,-,,y\boxed{\textcircled{v}},\frown y+1\textcircled{v}}{\boxed{\textcircled{v}}y,\frown y+1\boxed{\textcircled{v}}}$.

6 summa *(1)* si ad $\dfrac{\mathrm{y}^2+2\mathrm{y}}{\mathrm{y}^5+2\mathrm{y}^4+\mathrm{y}^3}$ adiecissem *(2)*. Tractabiliores *L*   7 f. $\dfrac{1}{\mathrm{y}^2-\mathrm{y}}$. *(1)* Eodem modo si un *(2)* Ex *L*   9 ducantur *(1)* summae *(2)* factis *L*   13 f. datur *(1)* quadratura, *(2)* summa *L*   17 f. decyphrandi. *(1)* Nunc tantum dicam: *(2)* Investigemus *L*

$\textcircled{v}$ significat regulam qua y tractanda est, $\boxed{\textcircled{v}}$ significat aliam regulam. Casus quosdam percurramus. Primus esto, cum nullus extat denominator, et fiet: $y\textcircled{v} - y + 1\textcircled{v}$. Eo casu $y\textcircled{v}$ vel continet signa radicalia in quibus sit y vel [non] continet, si nulla contineat signa radicalia in quibus sit y, tunc etiam differentia non habebit incognitam neque in denominatore, neque in vinculo, atque ita differentia componetur ex meris paraboloeidibus inter se compositis, quarum cum habebitur summa, non est ut huic casui immoremur; si radicem ingrediatur incognita et simplicem quidem v. g. si sit $\sqrt{y\textcircled{v}} - \sqrt{y+1,\textcircled{v}} \sqcap z$. fiet:

$y\textcircled{v} - 2\sqrt{y\textcircled{v},, \frown y+1,\textcircled{v}} + y+1,\textcircled{v} \sqcap z^2$, sive $-2\sqrt{\ldots} \sqcap z^2, -y\textcircled{v}, -y+1,\textcircled{v}$. Unde fiet:

$4, \frown y\textcircled{v} \frown y+1\textcircled{v} \sqcap z^4 - 2z^2 y\textcircled{v} - 2z^2 \frown y+1\textcircled{v} + y\textcircled{v},^2 + 2y\textcircled{v}, \frown y+1\textcircled{v}, + y+1,\textcircled{v}^2$.

Caeterum ut $\textcircled{v}$, non nihilo distinctius explicetur, ponendum est notatos esse a nobis casus, quibus tractatio variat, ut cum y est in denominatore, et cum est in vinculo, caetera explicabuntur per expressas potestates, v. g.

$ay^3 + by^2 + cy + d + n\sqrt{ey^3 + fy^2 + gy + h} + p\sqrt{ky^2 + ly + m} + q\sqrt{ry + s} \sqcap x$.

Unde pro differentia ponendo $y + \beta$ in locum y habetur differentia generalis, et nunc has nunc illas literas, ponendo aequales nihilo, aut datae quantitati, variae figurae aut series speciales habentur quarum data sit series; vel quadratura.

Sed ne prolixo nimis calculo nos induamus suffecerit neglectis caeteris, hanc sumi seriem unius tantum irrationalis, et denominatore carentem nempe:

$$\left.\begin{array}{lllll} \boxed{-ay^3} & \boxed{-\ by^2} & \boxed{-cy} & \boxed{-d} & -n\sqrt{\ ey^3\ +\ fy^2 + gy + h} \\ \boxed{+ay^3} & \boxed{+\ by^2} & \boxed{+cy} & \boxed{+d} & +n\sqrt{\ ey^3\ +\ fy^2 + gy + h} \\ ..3ay^2\beta & +2b\beta y & +c\beta & & +\quad 3ey^2\beta + 2f\beta y + g\beta \\ ..3a\beta^2 y & +\ b\beta^2 & & & \quad\ \ 3ey\beta^2 +\ f\beta^2 \\ ..\ \ a\beta^3 & & & & \quad\ \ e\beta^3 \end{array}\right\} \sqcap z.$$

Hinc statim patet, universaliter verum esse in figuris geometricis, quod termini in quibus β assurgit ad quadratum et ultra reici possint: nam si dicas fieri posse, ut servari

3 non *erg. Hrsg.* 4 in ... y *erg. L* 13 nihilo *(1)* rectius *(2)* distinctius *L* 13 est *(1)* totos *(2)* notatum *(3)* notatos *L* 16 v. g. *(1)* omnia signa y^3 + *(2)* ay^3 *L* 24+27 $\sqcap$ z. *(1)* Subscribamus denominatorem sed radice carentem: *(2)* Hinc *L* 28 β *(1)* excedit *(2)* assurgit *L*

debeant β^2, quia omnes β simplices evanescant; respondeo tunc a. b. c. fore, quodlibet $\sqcap$ 0. ergo et β^2 et β^3 evanescere. In seriebus autem arithmeticis servari debent. Nunc [ordinanda] aequatio est, ut tolli possint radices, sed antequam hoc faciamus, fingamus simplicioris calculi causa, primum a. b. esse nihil, fiet:

$$\begin{array}{llll}
\boxed{n^2ey^3}\boxed{+n^2fy^2}\boxed{+n^2gy}\boxed{+n^2h} & +2n^2ey^3 \quad +2n^2fy^2 \quad +2n^2gy+2n^2h & -z^2 & \sqcap \\
 & n^2 3ey^2\beta+n^2f\ \ \beta y+n^2g\ \ \beta & +2c\beta z & \\
 & n^2 3ey\beta^2+n^2f\ \ \beta^2 & -\beta^2c^2 & \\
 & n^2e\ \ \beta^3 & &
\end{array}$$

$$+2n^2\sqrt{\begin{array}{ll||ll||ll||ll}
\boxed{e^2y^6} & \boxed{efy^5} & \boxed{fey^5} & \boxed{f^2y^4} & \boxed{gey^4} & \boxed{gfy^3} & \boxed{hey^3} & \boxed{hfy^2} \\
\boxed{3e^2y^5\beta} & \boxed{2ef\beta y^4} & 3fey^4\beta & \boxed{+2f^2\beta y^3} & \boxed{3gey^3\beta} & \boxed{2gf\beta y^2} & \boxed{3hey^2\beta} & \boxed{+2hf\beta y} \\
\boxed{3e^2y^4\beta^2} & \boxed{\underline{ef\beta^2y^3}} & 3fey^3\beta^2 & \boxed{+\ \underline{f^2\beta^2y^2}} & \boxed{3gey^2\beta^2} & \underline{gf\beta^2y} & \boxed{3hey\beta^2} & \boxed{+\ \underline{hf\beta^2}} \\
\boxed{\underline{\underline{e^2y^3\beta^3}}} & \boxed{egy^4} & fe\beta^3y^2 & \boxed{fgy^3} & \boxed{\underline{\underline{gey\beta^3}}} & \boxed{g^2y^2} & \boxed{\underline{\underline{he\beta^3}}} & \boxed{+\ hgy} \\
 & \boxed{\underline{eg\beta y^3}} & & \boxed{\underline{fg\beta y^2}} & & \boxed{g^2\beta y} & & \underline{\boxed{hg\beta}} \\
 & \boxed{ehy^3} & & \boxed{fhy^2} & & \boxed{ghy} & & \boxed{h^2}
\end{array}}$$

Ergo quadrando prius aequationis latus, fiet:

2 f. debent. *(1)* Nunc ut per partes eamus *(2)* Nunc |ordinata *ändert Hrsg.*| *(a)* aequatione fiet: *(b)* aequatio *L*

$\boxed{4n^4e^2y^6}$	$\boxed{8n^4efy^5}$	$\boxed{8n^4egy^4}$	$-\ 4n^2z^2ey^3$	$\boxed{4n^4f^2y^4}$	$\boxed{8n^4fhy^2}$
$\boxed{12n^4e^2y^5\beta}$	$12n^4efy^4\beta$	$\boxed{12n^4egy^3\beta}$	$-\ 6n^2z^2ey^2\beta$	$+8n^4f^2y^3\beta$	$\boxed{8n^4fhy\beta}$
$\boxed{12n^4e^2y^4\beta^2}$	$12n^4efy^3\beta^2$	$\boxed{12n^4egy^2\beta^2}$	$-\ 6n^2z^2ey\beta^2$	$4n^4f^2y^2\beta^2$	$\underline{\underline{\boxed{4n^4fh\beta^2}}}$
$\underline{\boxed{4n^4e^2y^3\beta^3}}$	$\underline{\underline{4n^4efy^2\beta^3}}$	$\underline{\boxed{4n^4egy\beta^3}}$	$\underline{-\ 2n^2z^2e\beta^3}$	$4n^4f^2y^2\beta^2$	$-4n^2fy^2z^2$
$9n^4e^2y^4\beta^2$	$\boxed{8n^4efy^4\beta}$	$\boxed{4n^4egy^3}\beta$	$+\ 8n^2cey^3z\beta$	$\underline{4n^2f^2y\beta^3}$	$-4n^2fyz^2\beta$
$18n^4e^2y^3\beta^3$	$8\boxed{12}n^4efy^3\beta^2$	$6n^4egy^2\beta^2$	$+12n^2cey^2z\beta^2$	$\underline{n^4f^2\beta^4}$	$\underline{-2n^2fz^2\beta^2}$
$\underline{6n^4e^2y^2\beta^4}$	$12n^4efy^2\beta^3$	$6n^4egy\beta^3$	$+12n^2ceyz\beta^3$	$\boxed{8n^4fgy^3}$	$+8n^2fcy^2z\beta$
$9n^4e^2y^2\beta^4$	$\underline{4n^4efy\beta^4}$	$\underline{\underline{2n^4eg\beta^4}}$	$\underline{+\ 4n^2cez\beta^4}$	$\boxed{8n^4fgy^2\beta}$	$+8n^2fcyz\beta^2$
$\underline{6n^4e^2y\beta^5}$	$4n^4efy^3\beta^2$	$\boxed{8n^4ehy^3}$	$-\ 4n^2ec^2y^3\beta^2$	$\underline{4n^4fgy[\beta^2]}$	$\underline{+4n^2fcz\beta^3}$
$\underline{\underline{n^4e^2\beta^6}}$	$6n^4efy^2\beta^3$	$\boxed{12n^4ehy^2\beta}$	$-\ 6n^2ec^2y^2\beta^3$	$\boxed{4n^4fgy^2\beta}$	$-4n^2c^2fy^2\beta^2$
	$6n^4efy\beta^4$	$\boxed{12n^4ehy\beta^2}$	$-\ 6n^2ec^2y\beta^4$	$4n^4fgy\beta^2$	$-4n^2c^2fy\beta^3$
	$\underline{\underline{[2]n^4ef\beta^5}}$	$\underline{\underline{\boxed{4n^4eh\beta^3}}}$	$\underline{\underline{\underline{-\ 2n^2ec^2\beta^5}}}$	$\underline{\underline{2n^4fg\beta^3}}$	$\underline{\underline{\underline{-2n^2c^2f\beta^4}}}$

$\boxed{4n^4g^2y^2}$	$\underline{\boxed{4n^4h^2}}$	z^4
$\boxed{4n^4g^2y\beta}$	$-4n^2hz^2$	$-4z^3c\beta$
$\underline{\underline{n^4g^2\beta^2}}$	$+8n^2hcz\beta$	$\underline{+2z^2c^2\beta^2}$
$\boxed{8n^4ghy}$	$\underline{\underline{\underline{-4n^2hc^2\beta^2}}}$	$4c^2\beta^2z^2$
$\underline{\underline{\boxed{4n^4gh\beta}}}$		$\underline{-4c^3\beta^3z}$
$-4n^2gyz^2$		$+\beta^4c^4$
$\underline{-2n^2g\beta z^2}$		
$+8n^2cgyz\beta$		
$\underline{+4n^2cgz\beta^2}$		
$-4n^2c^2y\beta^2$		
$\underline{\underline{\underline{-2n^2c^2g\beta^3}}}$		

9 β^3 *L ändert Hrsg.* 12 2 *erg. Hrsg.*

Unde patet omnes terminos irrationales evanescere post quadrationem.

Et habetur tabula generalis serierum, et si omnia per β^2 et ultra multiplicata destruantur, figurarum quadrabilium, quae variant, pro varia explicatione cognitarum.

Termini pro figuris geometriae excerpti sequenti plagula parte schediasmatis huius quarta continebuntur. Omittendi et in quibus z. ascendit ultra quadratum.

Retinendi termini z^2. β^2. $z\beta$. NB. caeteris omnibus reiectis.

2 et ultra *erg. L* 5 ascendit *(1)* ad *(2)* ultra *L*

38_5. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS QUARTA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 8–9. 1 Bog. 2°. 4 S. — Auf Bl. 7 v° verworfene Nebenbetrachtung. 2/5 S.
Cc 2, Nr. 775 A tlw.

Pars IV^ta^ schediasmatis de seriebus et summis quadraturisque.

Ex tabula proposita excerpantur iam termini geometrici. Nullus autem reperitur terminus in quo sit z. simplex sine β. et β. simplex sine z: Ideo retentis terminis $z^2, \beta^2, z\beta$. caeteri omnes reiciantur, et pro β. substituatur a, velut unitas, vel etiam β^2, plane ascribi negligatur, fiet:

$$\left.\begin{array}{l} 9n^4e^2y^4 + 12n^4efy^3 + 6n^4egy^2 - 4n^2ec^2y^3 + 4n^4f^2y^2 \left\{\begin{array}{l} +[8]n^4fgy \\ -4\,n^2c^2gy \end{array}\right. - 4n^2c^2fy^2 \\ [-4n^2ec^2y^3] - 4n^2fz^2y^2 - 4n^2gyz^2 - 4n^2hz^2 + 8n^2cey^3z + 8n^2fcy^2z \\ +8n^2cgyz + 8n^2hcz + n^4g^2 - 4n^2hc^2 \end{array}\right\} \sqcap 0. ☽$$

Quod si iam ponamus $e \sqcap 0$. evanescet y^4. et y^3. et ponendo praeterea $n^4g^2 - 4n^2hc^2 \sqcap 0$. restabit:

$$4n^4f^2y^2 + \begin{array}{l}[8]n^4fg\,y \\ -4\,n^2c^2g..\end{array} - 4n^2c^2fy^2 - 4n^2fz^2y^2 - 4n^2gyz^2 - 4n^2hz^2 + 8n^2fcy^2z + 8n^2cgyz +$$

$8n^2hcz$. cui formulae rursus quaedam adimere possemus, ut ponendo $4n^4f^2y^2 - 4n^2fz^2y^2 \sqcap 0$. nullus restabit terminus y^2. Si vero potius f. ponas $\sqcap 0$. fiet:

$-4n^2c^2gy - 4n^2gyz^2 - 4n^2hz^2 + 8n^2cgyz + 8n^2hcz$. cuius figurae quadratrix erit:

$$cy + \sqrt{gy + \frac{g^2}{4c^2}} \sqcap x.$$

6 geometrici *(1)*, restabit: *(2)*. Nullus *L* 8 reiciantur, *(1)* ita fiet: *(2)* et *L* 8 unitas, *(1)* habebimus: *(2)* vel *L* 10 4 *L ändert Hrsg.* 11 $+4n^4ez^2y^3$ *L ändert Hrsg.* 11 ☽ *erg. L* 15 4 *L ändert Hrsg.* 18 f. fiet: *(1)* $-4n^2gyz^2 - 4n^2hz^2 + 8n^2cgyz + 8n^2hcz$. et divisis omnibus per $4n^2z$, restabit: $-gyz - hz + 2cgy + 2hc \sqcap 0$. quae aequatio est ad hyperbolam, et quia *(a)* g^2, posuimus *(b)* h posuimus $\sqcap \frac{g^2}{4c^2}$, substituendo eius valorem: fiet $-\not{g}yz - \frac{g^{\not{2}}}{4c^2}z + 2c\not{g}y + \frac{2g^{\not{2}}}{4c} \sqcap 0$. Unde si nullus insit error calculo habebitur quadratura hyperbolae. *(2)* $-4n^2c^2gy$ *L*

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11 ☽: Von hier Verweisstrich zu S. 420 Z. 1 – S. 422 Z. 2. 19 quadratrix: Leibniz setzt wie in der Lesart zu Z. 18 f. $h = \frac{g^2}{4c^2}$ sowie $n = 1$.

Quod an verum sit statim examinare possumus. Nam differentia duarum x, erit:

$$\boxed{cy} + c\beta + \sqrt{gy + g\beta + \frac{g^2}{4c^2}}\,\boxed{-cy} - \sqrt{gy + \frac{g^2}{4c^2}} \sqcap z.$$

Unde $2gy + g\beta + \frac{2g^2}{4c^2}\boxed{+gy}\boxed{+\frac{g^2}{c^2}} - z^2 + 2c\beta z - c^2\beta^2 \sqcap 2\sqrt{\boxed{g^2y^2}\boxed{+g^2\beta\ \ y}\boxed{+\frac{g^3\beta}{4c^2}}\left[\boxed{+\frac{2g^3}{4c^2}}\,..\,\boxed{+\frac{g^4}{16c^4}}\right]}$

$$\sqcap - \frown \left\{ \begin{array}{lll} [z] & \frown 1, \square,, & +2gy + g\beta. \\ -[c\beta] & & \frac{2g^2}{4c^2} \end{array} \right.$$

Et quadrando posterior aequationis pars dabit:

$$\cancel{z^4} - \cancel{4z^3c\beta} + \cancel{6z^2c^2\beta^2} - \cancel{4zc^3\beta^3} \cancel{+c^4\beta^4}, \quad -4gy \frown \begin{array}{l} z^2 \\ -2c\beta z \\ +c^2\beta^2 \end{array} \quad \cancel{-2g\beta} \frown \begin{array}{l} z^2 \\ \cancel{-2c\beta z} \\ \cancel{+c^2\beta^2} \end{array}$$

$$-\frac{4g^2}{4c^2} \frown \begin{array}{l} z^2 \\ -2c\beta z \\ +c^2\beta^2 \end{array} \quad \boxed{+4g^2y^2} \quad \begin{array}{ll} \boxed{+4g^2\beta\ \ y} & +g^2\beta^2\,; \\ \boxed{+8\frac{g^3}{[4]c^2}\,..} & \boxed{+4\frac{g^3\beta}{[4]c^2}} \\ & \boxed{+\frac{4g^4}{[16]c^4}} \end{array}$$

et delendo $+g^2\beta^2 - \frac{4g^2}{4c^2} \frown c^2\beta^2$. quae figura est rationalis simul ac quadrabilis et dabit: $y \sqcap \dfrac{-\frac{g^2}{c^2}z^2 + \frac{2g^2}{c}z}{z - c, \square,, \frown [4g]}$, et [dividendo] per $\dfrac{g^2}{c} \smile [4g]$. fiet $y \sqcap \dfrac{-\frac{z^2}{c} + 2z}{z - c, \square}$, et multiplicando

4 $\boxed{+\frac{2g^2}{c^2}}\,..\,\boxed{+\frac{g^4}{4c^4}}$ *L ändert Hrsg.* 5 z^2 *L ändert Hrsg.* 6 $c\beta z$ *L ändert Hrsg.* 12 4 *erg. Hrsg. zweimal* 13 16 *erg. Hrsg.* 14 delendo *(1)* $+g^2\beta^2$, contra *(2)* $+g^2\beta^2 - \frac{4g^2}{4c^2} \frown c^2\beta^2$ *(a)* divisisque omnibus *(b)*. quae *L* 15 $-4g$ *L ändert Hrsg. zweimal* 15 multiplicando *L ändert Hrsg.*

per c, fiet: $y \sqcap \dfrac{-z^2+2cz}{z-c,\square}$, auferatur $\dfrac{c^2}{z-c,\square}$, quae figura est quadrabilis. Iam quadrabilis addita quadrabili dat quadrabilem, erit ergo quadrabilis: $y \sqcap \dfrac{-z^2+2cz-c^2}{z-c,\square}$, et fit $y \sqcap -1$. Unde patet nos nihil magnopere egisse quadrando hanc figuram cum sit hyperbola cubica. Calculi interea veritatem quodammodo comprobavimus.

Aliam eodem plane modo formulam fieri operae pretium est pro seriebus irrationali carentibus, sed contra habentibus fractionem; ubi hoc etiam commodi habebimus, quod litera z. non assurget ultra simplicem exponentem.

$$\left.\begin{array}{ll} -\ cy^4-\ dy^3-\ ey^2-fy-g,\smile & hy^4+\ ly^3+\ my^2+ny+p \\ +\ cy^4+\ dy^3+\ ey^2+fy+g\ \smile & hy^4+\ ly^3+\ my^2+ny+p \end{array}\right\} \sqcap z.$$

$$\begin{array}{ll} 4\ c\beta y^3+\ 3d\beta y^2+2e\beta y\ +f\beta & 4h\beta y^3+\ 3l\beta y^2+2m\beta y\ +n\beta \\ 6c\beta^2y^2+3d\beta^2y\ +e\beta^2 & 6h\beta^2y^2+3l\beta^2y\ +m\beta^2 \\ 4c\beta^3y\ +\ d\beta^3 & 4h\beta^3y\ +\ l\beta^3 \\ +c\beta^4 & +\ h\beta^4 \end{array}$$

Et multiplicando per crucem, fiet:

chy^8	cly^7	cmy^6	cny^5	cpy^4
$4ch\beta y^7$	$\boxed{4}cl\beta y^6$	$2\boxed{4}cm\beta y^5$	$3\boxed{4}cn\beta y^4$	$4cp\beta y^3$
$6ch\beta^2y^6$	$6cl\beta^2y^5$	$3\boxed{6}cm\beta^2y^4$	$6cn\beta^2y^3$	$6cp\beta^2y^2$
$4ch\beta^3y^5$	$4cl\beta^3y^4$	$4cm\beta^3y^3$	$4cn\beta^3y^2$	$4cp\beta^3y$
$ch\beta^4y^4$	$cl\beta^4y^3$	$cm\beta^4y^2$	$cn\beta^4y$	$cp\beta^4$
$\boxed{dhy^7}$	$\boxed{dly^6}$	$\boxed{dmy^5}$	$\boxed{dny^4}$	$\boxed{dpy^3}$
$\boxed{3dh\beta y^6}$	$\boxed{3dl\beta y^5}$	$\boxed{3}dm\beta y^4$	$2\boxed{3}dn\beta y^3$	$3dp\beta y^2$
$\boxed{3dh\beta^2y^5}$	$3dl\beta^2y^4$	$3dm\beta^2y^3$	$3dn\beta^2y^2$	$3dp\beta^2y$
$dh\beta^3y^4$	$dl\beta^3y^3$	$dm\beta^3y^2$	$dn\beta^3y$	$dp\beta^3$
$\boxed{ehy^6}$	$\boxed{ely^5}$	$\boxed{emy^4}$	$\boxed{eny^3}$	$\boxed{epy^2}$
$\boxed{2eh\beta y^5}$	$\boxed{2el\beta y^4}$	$2\boxed{em\beta y^3}$	$\boxed{2}en\beta y^2$	$2ep\beta y$

1 $\dfrac{-z^2+2cz}{z-c,\square}$, *(1)* et mutatis signis: $y \sqcap \dfrac{z \frown z-c}{z-c,\square}$. Ergo $y \sqcap \dfrac{z}{z-c}$: *(2)* addatur *(3)* auferatur *L*

3 f. cum ... cubica *erg. L* 5 pro *(1)* curvis ratio *(2)* seriebus *L*

$eh\beta^2y^4$	$el\beta^2y^3$	$em\beta^2y^2$	$en\beta^2y$	$ep\beta^2$
fhy^5	fly^4	fmy^3	fny^2	fpy
$fh\beta y^4$	$fl\beta y^3$	$fm\beta y^2$	$fn\beta y$	$fp\beta$
ghy^4	gly^3	gmy^2	gny	gp
$-\ hcy^8$	$-\ hdy^7$	$-\ hey^6$	$-\ hfy^5$	$-\ hgy^4$
$-\ 4hc\beta y^7$	$-\ \boxed{4}hd\beta y^6$	$-\ 2\boxed{4}he\beta y^5$	$-\ 4hf\beta y^4$	$-\ 4hg\beta y^3$
$-\ 6hc\beta^2y^6$	$-3\boxed{6}hd\beta^2y^5$	$-\ 6he\beta^2y^4$	$-\ 6hf\beta^2y^3$	$-6hg\beta^2y^2$
$-\ 4hc\beta^3y^5$	$-3\boxed{4}hd\beta^3y^4$	$-\ 4he\beta^3y^3$	$-\ 4hf\beta^3y^2$	$-4hg\beta^3y$
$-\ hc\beta^4y^4$	$-\ hd\beta^4y^3$	$-\ he\beta^4y^2$	$-\ hf\beta^4y$	$-\ hg\beta^4$
$-\ lcy^7$	$-\ ldy^6$	$-\ ley^5$	$-\ lfy^4$	$-\ lgy^3$
$-\ 3lc\beta y^6$	$-\ 3ld\beta y^5$	$-\ \boxed{3}le\beta y^4$	$-\ 2\boxed{3}lf\beta y^3$	$-\ 3lg\beta y^2$
$-\ 3lc\beta^2y^5$	$-\ 3ld\beta^2y^4$	$-\ 3le\beta^2y^3$	$-\ 3lf\beta^2y^2$	$-\ 3lg\beta^2y$
$-\ lc\beta^3y^4$	$-\ ld\beta^3y^3$	$-\ le\beta^3y^2$	$-\ lf\beta^3y$	$-\ lg\beta^3$
$-\ mcy^6$	$-\ mdy^5$	$-\ mey^4$	$-\ mfy^3$	$-\ mgy^2$
$-\ 2mc\beta y^5$	$-\ 2md\beta y^4$	$-\ 2me\beta y^3$	$-\ \boxed{2}mf\beta y^2$	$-\ 2mg\beta y$
$-\ mc\beta^2y^4$	$-\ md\beta^2y^3$	$-\ me\beta^2y^2$	$-\ mf\beta^2y$	$-\ mg\beta^2$
$-\ ncy^5$	$-\ ndy^4$	$-\ ney^3$	$-\ nfy^2$	$-\ ngy$
$-\ nc\beta y^4$	$-\ nd\beta y^3$	$-\ ne\beta y^2$	$-\ nf\beta y$	$-\ ng\beta$
$-\ pcy^4$	$-\ pdy^3$	$-\ pey^2$	$-\ pfy$	$-\ pg$

Dividenda per factum ex duobus denominatoribus:

h^2y^8	lhy^7	mhy^6	nhy^5	phy^4
$4h^2\beta y^7$	$4lh\beta y^6$	$4mh\beta y^5$	$4nh\beta y^4$	$4ph\beta y^3$
$6h^2\beta^2y^6$	$6lh\beta^2y^5$	$6mh\beta^2y^4$	$6nh\beta^2y^3$	$6ph\beta^2y^2$
$4h^2\beta^2y^5$	$4lh\beta^3y^4$	$4mh\beta^3y^3$	$4nh\beta^3y^2$	$4ph\beta^3y$
$h^2\beta^4y^4$	$lh\beta^4y^3$	$mh\beta^4y^2$	$nh\beta^4y$	$ph\beta^4$
hly^7	l^2y^6	mly^5	nly^4	ply^3
$3hl\beta y^6$	$3l^2\beta y^5$	$3ml\beta y^4$	$3nl\beta y^3$	$3pl\beta y^2$
$3hl\beta^2y^5$	$3l^2\beta^2y^4$	$3ml\beta^2y^3$	$3nl\beta^2y^2$	$3pl\beta^2y$

$$\begin{array}{rrrrr}
\underline{hl\beta^3y^4} & \underline{l^2\beta^3y^3} & \underline{ml\beta^3y^2} & \underline{nl\beta^3y} & \underline{pl\beta^3} \\
hmy^6 & lmy^5 & m^2y^4 & nmy^3 & pmy^2 \\
2hm\beta y^5 & 2lm\beta y^4 & 2m^2\beta y^3 & 2nm\beta y^2 & 2pm\beta y \\
\underline{hm\beta^2y^4} & \underline{lm\beta^2y^3} & \underline{m^2\beta^2y^2} & \underline{nm\beta^2y^3} & \underline{pm\beta^2} \\
hny^5 & lny^4 & mny^3 & n^2y^2 & pny \\
\underline{hn\beta y^4} & \underline{ln\beta y^3} & \underline{mn\beta y^2} & \underline{n^2\beta y} & \underline{pn\beta} \\
hpy^4 & lpy^3 & mpy^2 & npy & p^2
\end{array}$$

Unde iam excerpendo eas tantum quantitates, quae ad figuras geometricas notandas servire possint, et proinde abiectis illis omnibus in numeratore, in quibus β. assurgit ad quadratum et ultra, et abiectis omnibus in nominatore in quibus est β, fiet formula reformata $\odot$.

$$\left.\begin{array}{ccccccccc}
 & cly^6 & +2cm\,y^5 & 3nc\,y^4 & 4cp\,y^3 & & & & \\
 & -hd\,.. & & dm\,.. & 2dn.. & 3dpy^2 & & & \\
 & & -2he\ .. & -el\,.. & * & en.. & 2epy & & \\
 & & & [-3fh..] & -2fl\,.. & -mf.. & * & fp & \\
 & & & & -4gh.. & -3gl\,.. & [-]2gm\ .. & [-]ng & \\
\hline
h^2y^8 & 2\,hl\ y^7 & 2hmy^6 & 2hny^5 & 2hpy^4 & & & & \\
 & & l^2.. & 2lm.. & 2ln\ .. & 2pl\ y^3 & & & \\
 & & & & m^2\ .. & 2mn\ .. & 2mpy^2 & & \\
 & & & & & & n^2.. & 2npy & \\
 & & & & & & & & p^2
\end{array}\right\}\sqcap\ z.\ \odot$$

12–21 Nota huius seriei progressus in numeratore multipli ipsarum $c.\,d.\,e.\,f.\,g.$ per numeros arithmeticae progressionis multiplicati sunt, v. g. multipli ipsius e. per $-2-1$ ǂ $0+1+2$.
Videndum an inde regula duci possit, quae serviat ad regressum item, an etiam ad series arithmeticas quiddam tale applicare liceat, quia illis regula Slusiana accommodari non potest.

9 omnibus | in *(1)* denominatore *(2)* numeratore *erg.* |, in *L* 10 ultra, *(1)* fiet *(a)* regula reformat *(b)* formula reformata ⊙ *(2)* et *L* 15 −3fh.. *erg. Hrsg.* 16 − *erg. Hrsg. zweimal*

Figurae ergo omnes, quarum aequatio ex cognitarum huius formulae generalis explicatione nasci potest, quadrari possunt. Possunt autem explicationes ita institui, ut numerator et nominator per divisores communes deprimantur.

Sed et termini quidam literaeve quaedam possunt poni nihilo aequales vel quantitati datae. Si e. g. ponas h, et l et $c \sqcap 0$. et $m \sqcap 0$. [fiet:]

$$\frac{\boxed{dmy^4} + 2dny^3 + \begin{matrix} 3dp \\ en \\ \boxed{+\ mf} \end{matrix} y^2 + \begin{matrix} 2ep \\ \boxed{[-]2gm} \end{matrix} y \begin{matrix} fp \\ [-ng] \end{matrix}}{n^2y^2 + 2npy + p^2} \sqcap z.$$

Tentandum ergo an ita explicari possit numerator iste, ut dividi possit per $ny + p$, quo dividi potest nominator,

9 NB. quando geometrice quaeruntur differentiae, semper notandum est in denominatore differentiae, sumendum tantum nominatoris figurae cuius differentiae sunt quadratum.

1 omnes, *(1)* quae huius formul *(2)* quarum *L* 1 f. explicatione *(1)* fieri *(2)* nasci *L* 5–9 et l *(1)* ⊓ 0. *(2)* et c ⊓ 0. *(a)* fiet: *(b)* et m ⊓ 0. | fiet: *erg. Hrsg.* | ... ⊓ z. *L* 7 + *L ändert Hrsg.* 7 ng *L ändert Hrsg.* 14 nominatoris *erg. L*

415,26 regula Slusiana: s. *An Extract of a Letter from the Excellent Renatus Franciscus Slusius*, *Philosophical Transactions* VII Nr. 90 vom 20./30. Jan. 1672/1673 S. 5143–47 (Nachtrag in VIII Nr. 95 vom 23. Juni/3. Juli 1673 S. 6059). Leibniz hat einen Auszug aus diesem Artikel angefertigt, *Methodus ducendi tangentes ad omnis generis curvas* (Cc 2, Nr. 616). 5 Si ... ponas: Leibniz vereinfacht in zwei Schritten. Im ersten setzt er $h = l = c = 0$ und schreibt die Gleichung an. Im zweiten setzt er auch $m = 0$ und umrahmt die zu eliminierenden Terme.

$$\begin{array}{llll} 2\cancel{dny^3}+3\cancel{dpy^2} & +\cancel{2epy} & fp & ⨍ \quad 2dy^2\,\dfrac{\left\{\begin{array}{l} +\textcircled{3}dpy \\ +\ \ en.. \\ \boxed{-\ 2dp..} \end{array}\right.}{n}\ \dfrac{\left\{\begin{array}{l} +\textcircled{2}epn \\ -\ \ dp^2 \\ \boxed{-\ \ enp} \end{array}\right.}{n^2} \\ \quad\ \cancel{+\ en..} & & [-]ng & \end{array}$$

$$\begin{array}{|c|} \hline ny + p \\ \hline 2\cancel{dny^3}+2\cancel{dpy^2} \\ \hline \end{array}$$

$$\begin{array}{l} \cancel{+2dp..} \\ \cancel{-2dp..} \end{array}$$

$$\begin{array}{|ll|} \hline ny\ + & p \\ \hline \begin{array}{l} \cancel{3dpy^2} \\ +\ \cancel{en}.. \\ \cancel{-2dp..} \end{array} & \dfrac{\left\{\begin{array}{l} \textcircled{3}\,\cancel{dp^2y} \\ \cancel{+enp..} \\ \boxed{-\ 2dp^2..} \end{array}\right.}{n} \\ \hline \end{array}$$

$$\dfrac{\left\{\begin{array}{l} +\ \cancel{dp^2y} \\ \cancel{+enp..} \end{array}\right.}{\cancel{n}}$$

$$\cancel{\dfrac{\left\{\begin{array}{l} -\ dp^2.. \\ -\ enp.. \end{array}\right.}{n}}$$

$$\begin{array}{cc} \hline ny & +\ \ p \\ \hline \cancel{\dfrac{\left\{\begin{array}{l} epn.. \\ dp^2.. \end{array}\right.}{n}} & \dfrac{\left\{\begin{array}{l} ep^2n \\ -\ dp^3 \end{array}\right.}{n^2} \\ \hline \end{array}$$

Unde iam patet ut divisio exacte procedere possit, tantum poni debere

2 $-$ *erg. Hrsg.*

1–21 Leibniz markiert die Streichungen im zweiten Rechenschritt teilw. mit hier nicht wiedergegebenen Zählstrichen.

$f\,\boxed{p} \sqcap \dfrac{ep^2n - dp^3[+]n^3g}{n^2p}$ et fiet: $\dfrac{2dn^2y^2 + dpn\,y + epn + en^2\ ..\ - dpp}{[n^2, \frown\ ny + p]} \sqcap z$. Quae aequatio est ad hyperbolam, siquidem ulterius deprimi non potest, quod superest ut experiamur, hoc modo:

$$\begin{array}{llll}
2dn^2y^2 \quad dpn\,y & \boxed{epn} & ⨍ & +2dny + dp + en - 2dp \\
\quad\ en^2\ .. & -dp^2 & & \\
\hline
\boxed{ny + p} & & & \\
\quad -2dnp\ .. & & & \\
\quad \boxed{n\,y + \quad p} & & & \\
\qquad -dp^2 & & & \\
\qquad \boxed{-enp} & & & \\
\qquad +2dp^2 & & & \\
\end{array}$$

2 f $\boxed{\text{p}}$ ⊓ *(1)* $\dfrac{\text{ep}^2\text{n} - \text{dp}^3}{\text{n}^2\text{p}}$, sive fn ⊓ en−dp sive f ⊓ $\dfrac{\text{en} - \text{dp}}{\text{n}}$ et fiet: 2dy^2+ *(2)* $\dfrac{\text{ep}^2\text{n} - \text{dp}^3 |\ -\ \textit{ändert Hrsg.}\ | \text{n}^3\text{g}}{\text{n}^2\text{p}}$ *(a)* et ponendo e *(b)* et *L* 2 $\text{n}^2\text{y} + \text{p}$ *L ändert Hrsg.* 4 +en *(1)* −2dnp *(2)* −2dp *L* 11 *(1)* $+2\text{dnp}^2$ *(2)* $+2\text{dp}^2$ *L*

Unde patet divisionem procedere et fieri aequationem ad lineam rectam, quare nihil inde pro hyperbola duci potest.

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1 *Kontrollbetrachtung zur Lesart, nicht gestrichen*:

Methodo Slusii $$3dy^2t + 2eyt\,\frac{\left\{\begin{array}{l} enp^2\ t\ \boxed{+\,g} \\ -\ dp^3\,.. \\ +n^2g\ .. \end{array}\right.}{np} \sqcap my^2x + nyx + px$$

ponendo t. intervallum tangentis ab ordinata. Iam ut t ad x, ita β ad z. Erit $z \sqcap \dfrac{tx}{\beta}$.

Iam $t \sqcap \dfrac{my^2x + nyx + px}{3dy^2 + 2ey\,\dfrac{\left\{\begin{array}{l} +enp^2 \\ -dp^3 + n^2g \end{array}\right.}{np}}$. Ergo $z \sqcap \dfrac{my^2x^2 + nyx^2 + px^2}{3dy^2 + 2ey\,\dfrac{\left\{\begin{array}{l} +enp^2 \\ -dp^3 + n^2g \end{array}\right.}{np}}$. pro x. ponendo

eius valorem $$x \sqcap \frac{dy^3 + ey^2\,\dfrac{\left\{\begin{array}{l} enp^2\,y\ +\ g \\ -\ dp^3\,.. \\ +n^2g\ .. \end{array}\right.}{n}}{my^2 + ny + p}.$$

Sed haec quo minus fallamur summa cum exactitudine resumi operae pretium est.

1 divisionem *(1)* universaliter non procedere, nisi sit aequatio inter $-\mathrm{dp}^2$ et $\mathrm{dp}^2 - 2\mathrm{dnp}^2$ seu inter 1 et n. Quodsi ergo nullus inest error calculo, sequetur hinc absoluta hyperbolae quadratura. Quod facile experiri licet methodo Slusii. *(2)* procedere *L* 8 ponendo . . . ad z. *erg. L*

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4 Methodo Slusii: s. o. Erl. zu S. 415 Z. 26. — Leibniz vergißt auf der linken Seite der Tangentengleichung die Terme $-2mxyt - nxt$ und übersieht außerdem, daß er zuvor bereits $m = 0$ gesetzt hat. In Z. 7 müßte $z = \dfrac{\beta x}{t}$ stehen, der Fehler beeinträchtigt die weitere Kontrollrechnung zusätzlich. Ab Z. 8 wird β als „unitas constructionis“ gleich 1 gesetzt; vgl. S. 411 Z. 8. Leibniz bricht schließlich ab und nimmt die Betrachtung in N. 38_7 S. 432 Z. 10 wieder auf.

Tentandum an aequatio ad ellipsin vel hyperbolam $-2az \mp \frac{a}{b}z^2 + y^2, \sqcap 0.$ per aliam

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1 *Nebenbetrachtung zur Lesart, nicht gestrichen*: $y^2 + z^2 - 2az \sqcap 0.$ ducatur in $\frac{p}{a}y + qz + r.$

1–421,3 *Nebenbetrachtung auf Bl. 7 v°, verworfen*:
Nihil deest.

$a^2 + \frac{a}{b}\omega^2 \sqcap z^2$. Ergo $p^2 \sqcap a^2 + \frac{a}{b}\omega^2 + v^2$ vel $v^2 + \frac{a}{b}\omega^2 \begin{array}{l} + a^2 \\ - p^2 \end{array} \sqcap 0.$ et faciendo

$$v \sqcap x + c, \text{ et } \omega \sqcap y + d. \text{ fiet: } x^2 + 2cx + \boxed{c^2} + \frac{a}{b}y^2 + \frac{2a}{b}dy \boxed{+\frac{a}{b}d^2} \begin{array}{l} + \ a^2 \\ - \ p^2 \\ + \ c^2 \\ + \frac{a}{b}d^2 \end{array} \sqcap 0.$$

Multiplicetur haec aequatio per aliam, scilicet per $x + \frac{r}{a}y - s \sqcap 0.$ fiet:

$$\begin{array}{llllllll} \frac{r}{b}y^3 & + \ x^3 & + \ \frac{r}{a}x^2y & + \ \frac{a}{b}y^2x & + \ 2cx^2 & + \ \frac{2rd}{b}y^2 & + \ \frac{2ad}{b}xy & \sqcap 0. \\ & & & & - \ s.. & - \ \frac{a}{b}s \ .. & \ \ \frac{2rc}{a}.. & \end{array}$$

$$a\odot\left\{\begin{array}{l} + \ a^2x \\ - \ p^2.. \\ + \ c^2.. \\ + \frac{a}{b}d^2.. \end{array}\right. \qquad r\odot\left\{\begin{array}{l} \quad ray \\ - \ \frac{r}{a} \ p^2.. \\ + \ \frac{r}{a} \ c^2.. \\ + \ \frac{r}{b} \ d^2.. \end{array}\right. \qquad \odot as\left\{\begin{array}{l} - \ sa^2 \\ + \ sp^2 \\ - \ sc^2 \\ - \frac{sa}{b}d^2 \end{array}\right.$$

$$- \ 2cs.. \qquad\qquad -\frac{2a}{b}ds \ ..$$

1 ad *(1)* circulum $2az - z^2$ *(2)* ellipsin *L*

---

1 Tentandum: Von hier Verweisstrich auf die Gleichung ☽ S. 411 Z. 10–12. — Vgl. auch *Schediasma de superficiebus conoeidum et sphaeroeidum. Item de curva ellipsis et hyperbolae* (Cc 2, Nr. 773), datiert 3. Okt. 1674.

quandam multiplicata, qualis est $py^2 + qy + s$, reddi possit similis propositae formulae generali ☽, fiet enim

$$+\,py^4 \mp \frac{pa}{b}y^2z^2 - 2apzy^2 + qy^3 \mp \frac{aq}{b}yz^2 - 2aqyz + sy^2 \mp \frac{a}{b}sz^2 - 2azs \sqcap 0.$$

Sed video addendum esse $+y^3$. et aequationem ad hyperbolam vel ellipsin ponendam esse completam: $y^2 + bz^2 + cz + dy + e \sqcap 0$. ducendam in $ty^3, + py^2 + qy + rz + s + \omega yz$. Sed quia ita inevitabiliter prodit ty^5, quae in formula generali non reperitur, ideo necesse est formulam generalem multiplicari per $ly + mz + n$. Sed cum semper quaedam difficultates sint obstiturae, ideo multiplicata formula generali per $ly + mz + n$. ut loca eius quantum licet repleantur; particularis ad conicam datam, multiplicanda erit per formulam completam; unde sub exitum calculi statim apparebit, quae elidi possint aut conferri.

Esto $x \sqcap \dfrac{bv^2}{a^2 \mp v^2}$, sive $a^2x \mp v^2x - bv^2 \sqcap 0$, explicatis x et v, et aequatione proveniente, multiplicata per $ly + mz + q + \omega yz$.

Prior autem explicata stabit: $a^2z \mp y^2z + byz + cy^2 + dy + e \sqcap 0$. quae ducta in $ly + mz + q + pyz$, dabit terminos omnes praeter y^4.

Generaliter omnes figurae quarum aequationes aut earum multipli, formulae huic aut eius multiplo aequari possunt sunt quadrabiles.

Exemplo etiam opus est formulae pro figuris aequationum affectarum quadrandis.

Ope formulae ☽, sed ipsis β^3 etc. nondum expunctis, haberi potest modus quo summae haberi possunt numerorum irrationalium finitorum pariter atque infinitorum, quod hactenus potuit nemo.

Inquirendum est in methodum similem a priori sive per synthesin, cuius ope appareat quaenam series habeant differentias in fine exhaustas, differentias inquam vel differentias differentiarum cuiuscunque gradus. Sane manifestum est ultimam differentiam esse rectangulo homogeneam. Ergo penultima, erit triangulo, antepenultima, parabolae, et

2 ☽ *erg. L* 7 +n. *(1)* quo facto *(2)* Sed *L* 12 et v, *(1)* ductisque omnibus in *(2)* ductaque *(3)* et *L* 17 f. quadrabiles. *(1)* Exemplum etiam opus est sive formulae pro aequationibus affectis quadrandis. Ope seriei ☽ habentur *(2)* Exemplo *L* 25 Ergo *(1)* praecedens erit *(2)* penultima *L*

22 Inquirendum: Die folgende Überlegung wird in N. 38₇ ab S. 434 Z. 6 wieder aufgenommen, das Schema S. 422 Z. 3–17 wird in erweiterter Form wiederholt.

caeterae paraboloeidibus, patet ergo hoc fieri non posse nisi in iis quae procedunt per potestates.

E				
a				
	F			
	$a-b$			
M		G		
b		$a-2b+c$		
	N		H	
	$b-c$		$a-3b+3c-d$	
		P		L
c		$b-2c+d$		$a-4b+6c-4d+e$
			Q	
	$c-d$		$b-3c+3d-e$	
d		$c-2d+e$		
	$d-e$			
e				

Iam si ponamus ipsam seriem primarum differentiarum E. F. G. H. L. procedere per potestates, videndum quales inde oriantur series, ut si series:

$a \quad a-b \quad a-2b+c \quad a-3b+3c-d \quad a-4b+6c-4d+e$ etc.

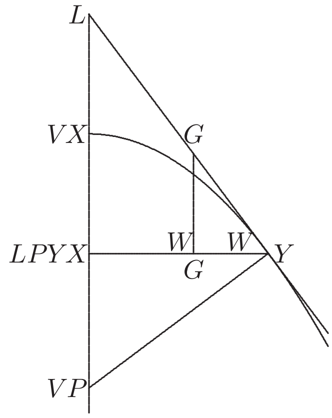
ponatur esse progressionis arithmeticae, ut sit $\boxed{a, -, a-b}$ $b \sqcap a-b, -a+2b-c$, fiet $c \sqcap 0$. ac proinde ista non procedunt in progressionibus decrescentibus puto tamen in ascendentibus. a ademta ab $a+b$, relinquit b, $a+b$ ademta ab $a+2b+c$, relinquit $b+c$. atqui non potest esse $b \sqcap b+c$. fiet enim rursus $c \sqcap 0$. Ideoque ista non procedunt: $a-a+b \sqcap b$. $\boxed{a}-b, \boxed{-a}+2b-c \sqcap b-c$. $\boxed{a}\boxed{-2b}\boxed{+c}\boxed{-a}+\boxed{3}b-2\boxed{3}c+d \sqcap b-2c+d$. Unde patet differentias seriei E. F. G. H. L. esse seriem M. N. P. Q.

38_6. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS QUARTA BIS

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 10–11. 1 Bog. 2°. 3 S. u. 4 Z. auf Bl. 11 v°. Überschrift ergänzt.
Cc 2, Nr. 775 A tlw.

P a r s q u a r t a s c h e d i a s m a t i s d e s e r i e b u s e t s u m m i s q u a d r a t r i c i b u s .

[*Teil 1*]



[*Fig. 1*]

Si in figura praesenti, curva X, Y ponatur esse parabola, cuius axis sit X, PYX, P, et aequatio: $2ax \sqcap y^2$. fiet: $2al \sqcap 2y^2$, aut $al \sqcap y^2$, aut $l \sqcap \frac{y^2}{a}$, aut $l \sqcap 2x$. et quia $pl \sqcap y^2$, erit $2px \sqcap y^2$, aut $2px \sqcap 2ax$, aut $p \sqcap a$.

Quid si iam propositum sit, investigare figuram in qua p, sit recta constans, a. ponendo $X, P \sqcap v$. erit $p \sqcap v - x \sqcap a$. fiet: $v^2 - 2vx + x^2 \sqcap a^2 \sqcap p^2$, et $y^2 \sqcap s^2 - a^2$. vel

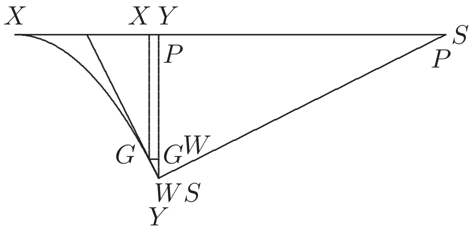
12 f. constans, a. *(1)* vocando X, P, *(2)* ponendo *L*

5 P a r s q u a r t a : Zählung wiederholt sich.

$y^2 \sqcap v^2 - 2vx + x^2 + s^2$. Haec novissima aequatio determinanda est ad duas radices aequales utriusque incognitae x. et s. Inde pro s. habebitur eius valor. Et vero id est in quaestione, an ab ista aequatione generali curvis omnibus communi inchoari possit calculus, eaque determinari ad duas utriusque incognitae radices aequales. Id vero fieri posse non puto.

Quando ergo quaestiones adeo simplices proponuntur, ut sint ne tractabiles quidem, mutandae sunt in compositiores. Porro $\dfrac{l}{y}$ seu $\dfrac{2x}{\sqrt{2ax}}$ est $\sqcap \dfrac{g}{w}$. Ergo $\dfrac{w}{\beta} \sqcap \dfrac{\sqrt{2ax}}{2x}$. sive $w^2 \sqcap \dfrac{2ax[\beta^2]}{4x^2}$, sive $w^2 \sqcap \dfrac{a\beta^2}{2x}$, sive pro w. ponendo z, figura ipsis w. homogenea erit: $2z^2x \sqcap a^3$.

Quod si iam quaeramus figuram, in qua w. valeat $\dfrac{\beta\sqrt{2ax}}{2x}$. seu $\dfrac{w}{\beta} \sqcap \dfrac{\sqrt{2ax}}{2x} \sqcap \dfrac{y}{l}$. Ergo $\dfrac{2ax}{4x^2} \sqcap \dfrac{y^2 \sqcap pl}{l^2}$. Unde $\dfrac{a}{2x} \sqcap \dfrac{p}{l}$. Sed satis ex his patet alia opus esse arte, ad regressum.



[*Fig. 2*]

8 β^2 *erg. Hrsg.* 10 $\dfrac{\beta\sqrt{2ax}}{2x}$. *(1)* erit $\dfrac{l}{y} \sqcap \dfrac{\sqrt{2ax}}{2x}$. sive $2xl \sqcap y\sqrt{2ax}$, sive $4x^2l^2 \sqcap 2y^2ax$. sive $2xl^2 \sqcap 2y^2a$. Iam $y^2 \sqcap pl$. fiet: $\not{2}xl \sqcap \not{2}ap$ *(2)* seu *L*

1 f. duas radices aequales: vgl. die Tangentenmethode von Descartes, *Geometria*, 1659, *DGS* I S. 40 bis 50 [Marg.] (vgl. *DO* VI S. 413–424). 7 $\dfrac{w}{\beta} \sqcap \dfrac{\sqrt{2ax}}{2x}$: Leibniz geht hier stillschweigend von g zu β über, in S. 425 Z. 3 setzt er sie explizit gleich. 12 *Fig. 2*: Leibniz vertauscht in der Fig. 2 gegenüber dem Vorhergehenden die x- und y-Achse, behält aber die Parabelgleichung $2ax = y^2$ unverändert bei. Dadurch wird die folgende Überlegung bis S. 425 Z. 12 beeinträchtigt.

Sumamus exemplum differentiarum inter ordinatas parabolae ad tangentem, $2ax \sqcap y^2$. Unde $2ax \sqcap 2yl$, sive $l \sqcap \frac{ax}{y}$. sive pro ax ponendo $\frac{y^2}{2}$ fiet: $l \sqcap \frac{y^2}{2y}$. Ergo $l \sqcap \frac{y}{2}$.

Iam $\frac{l}{x} \sqcap \frac{g \sqcap \beta}{w}$. Erit $w \sqcap \frac{x\beta}{l}$, sive $\frac{y^2\beta}{2al}$ sive $\frac{y^2\beta}{ay}$, sive $\frac{y\beta}{a} \sqcap w$. Triangulum ergo differentiis ordinatarum parabolae ad tangentem verticis homogeneum est. Sumta iam aequatione generali: $s^2 \sqcap v^2-2vx+x^2+y^2$, est autem $\frac{\not\beta a}{y\not\beta} \sqcap \frac{y}{v-x}$. erit $va-xa \sqcap y^2$, sive $v \sqcap \frac{y^2+xa}{a}$, et $v^2 \sqcap \frac{y^4+2y^2xa+x^2a^2}{a^2}$ et $a^2s^2 \sqcap y^4 \boxed{+2y^2xa} \boxed{+x^2a^2} \boxed{-2y^2xa} \boxed{-2x^2a^2} \boxed{+x^2a^2} +y^2a^2$ et fit: $s \sqcap \frac{y\sqrt{y^2+a^2}}{a}$. Sed quoniam hoc modo altera ex incognitis, evanescit, alia adhibenda est via, nimirum $G, YW, \sqcap \sqrt{\frac{\beta^2a^2+\beta^2y^2}{a^2}}$. Est autem $\frac{G, YW}{\beta} \sqcap \frac{s}{y}$. Ergo $s \sqcap \frac{y}{a}\sqrt{a^2+y^2}$, et $s^2 \sqcap \frac{y^2a^2+y^4}{a^2}$, et fiet ex aequatione generali, pro s^2 substituendo eius valorem, aequatio haec: $\boxed{y^2a^2} + y^4 \sqcap a^2v^2 - 2vxa^2 + x^2a^2 \boxed{+y^2a^2}$. et $y^2 \sqcap \mp av \pm ax$. Quoniam iam haec est methodus tangentium inversa, si multiplices y^2 per $\frac{1}{2}$, et x per 1, et av per 0, fiet: $\frac{y^2}{2} \sqcap ax$. sive $y^2 \sqcap 2ax$. Sed an hoc alias quoque succedat, videndum est. Et vero id non puto semper succedere posse.

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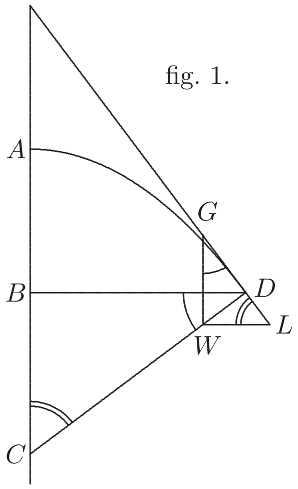
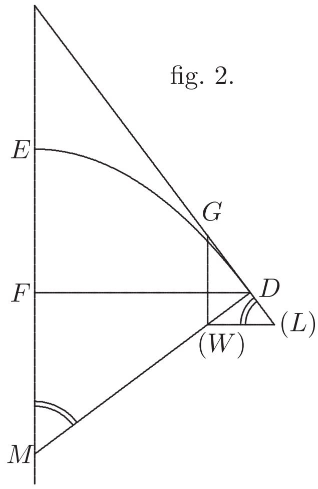
2 *Am Rande*: Iam $pl \sqcap x^2$. Ergo $\frac{p\not y}{2} \sqcap \frac{y^{\not 4 3}}{4a^2}$.



[*Fig. 3*]



[*Fig. 4*]

$\frac{BC}{BD} \sqcap \frac{WL}{GW}$. erit $BC \frown GW$ seu summa omnium BC, aequalis $BD \frown WL$ seu summae omnium BD ad basin. Summa autem omnium BD ad basin est ipsius maximae BD semiquadratum.

Porro manifestum est summam omnium WL esse ipsam maximam BD.

Proposita iam esto figura cuius ordinatae sunt e. g. $\sqrt{a^2 - x^2}$. Quaeritur figura in qua faciant functionem ipsarum BC, eius figurae ordinata erit summa omnium $\sqrt{a^2 - x^2}$.

Quaeritur et alia figura, in qua $\frac{\sqrt{a^2 - x^2}}{a}$ faciant functionem ipsarum WL. Summa earum erit FD ordinata figurae quae homogenea sit semiquadratis ipsarum BD.

Hinc patet momentum ipsarum BD ex axe, esse homogeneum figurae quadratrici ipsarum FD. Porro ut ad calculum veniamus, in fig. 1. patet esse: $\frac{BD}{\sqrt{a^2 - x^2}} \sqcap \frac{GW}{WL} \sqcap \frac{\beta}{WL}$. sive $BD \frown WL \sqcap \beta\sqrt{a^2 - x^2}$.

In aequatione generali: $BD^2 + BC^2 \sqcap CD^2$, vel $y^2 + BC^2 \sqcap p^2$, substituatur pro BC^2, eius valor, $a^2 - x^2$, fiet: $y^2 + a^2 - x^2 \sqcap p^2$. determinanda ad duas radices aequales, ut elidatur p.

6 $\sqrt{a^2 - x^2}$. *(1)* Quod si *(2)* Quaeritur *L*   10 momentum *(1)* figurae *(2)* ipsarum BD ex axe, esse *(a)* homogeneum figurae FD *(b)* homogeneum *(aa)* quadratrici ipsius figurae *(bb)* figurae *L*

In altera figura $\frac{GW}{WL}$ seu $\frac{a}{\sqrt{a^2-x^2}} \sqcap \frac{FD}{FM} \sqcap \frac{z}{FM}$. et erit $FM \sqcap z\frac{\sqrt{a^2-x^2}}{a}$. Eumque valorem inserendo in aequatione generali, fiet: $z^2 + \frac{z^2a^2 - z^2x^2}{a^2} \sqcap s^2$. ponendo $FD \sqcap z$. et $FM \sqcap s$. Est autem $z \sqcap \frac{y^2}{2a}$, et fiet: $\frac{y^4}{4a^2} + \frac{\frac{y^4a^2}{2a^2} - \frac{y^4x^2}{4a^2}}{a^2} \sqcap s^2$. sive $3a^2y^4 - y^4x^2 \sqcap 4s^2a^4$, et fiet: $2sa^2 \sqcap y^2\sqrt{3a^2-x^2}$.

Habemus quidem iam duas aequationes, ex quibus quaelibet est trium incognitarum, sed malum est, quod in iis duae etiam incognitae s. et p.

Methodus tangentium inversa, videtur esse eadem, cum methodo inveniendi planum quod superficiem cuiusdam solidi tangat in puncto dato. Ita enim fit, ut aequatio trium incognitarum determinetur ad duas, eo ipso, quia quaelibet ex incognitis duas habet radices aequales. Verum iam video discrimen aliquod subesse a methodo tangentium inversa, quia in ea tres quidem sunt incognitae, sed duae tantum ex illis habent duos valores aequales.

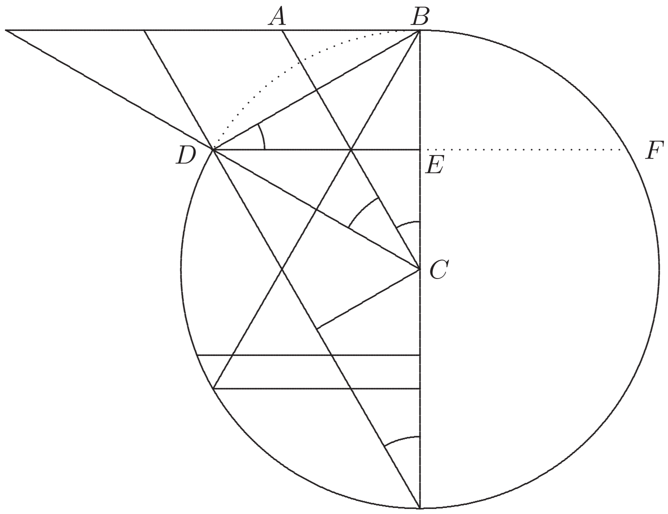
Methodus autem ducendi planum tangens ad superficiem, videtur coincidere cum methodo ducendi rectam tangentem ad curvam in solido quodam spatio sive extra planum constans divagantem; quam obiter innuit Cartesius, sub libri secundi finem.

Porro considerandum est has duas $s \sqcap MD$. et $p \sqcap CD$. plurimum habere affinitatis. Nondum tamen rationem invenio alterutram earum eliminandi. Nec vero hac quidem methodo facile eo perveniri posse arbitror.

2 f. ponendo ... FM $\sqcap$ s. *erg.* *L* 8 ut *(1)* duae *(2)* aequatio *(a)* duarum incognitarum *(b)* trium *L* 9 incognitis *(1)* du *(2)* tres ha *(3)* duas *L* 11 ea *(1)* duae *(2)* tres *L* 15 quam *(1)* sane curvam *(2)* obiter *L* 16 f. affinitatis *(1)*; quoniam etiam FD | fig. 2. *erg.* | sunt homogeneae ipsarum WL. fig. 1. scilicet $\frac{\mathrm{p}}{\mathrm{BC}} \sqcap \frac{\frac{\mathrm{p}}{\mathrm{WL} \frown \mathrm{a}}}{\beta}$ *(2)*. Nondum *L*

15 Cartesius: *Geometria*, 1659, *DGS* I S. 65 f. (vgl. *DO* VI S. 440 f.). 16 $s \sqcap MD$: Im Vorhergehenden benutzte Leibniz $s = FM$.

[*Teil 2*]



[*Fig. 5, teilweise in Blindtechnik*]

Triangula CBA, et DEB similia sunt. Angulus enim BDE insistens arcui BF, aequali arcui BD dimidius est anguli [BCD], ideoque aequalis angulo BCA.

4 est *(1)* arcus *(2)* anguli | BD *ändert Hrsg.* |, ideoque *L*

[*Teil 3*]

$$1\quad\underbrace{\frac{1}{3}\qquad\frac{1}{6}}_{\frac{1}{2}}^{2\text{ ad }1}\quad\underbrace{\frac{1}{10}\qquad\frac{1}{15}}_{\frac{5}{30}\left(\frac{1}{6}\right)}^{3\text{ ad }2}\quad\underbrace{\frac{1}{21}\qquad\frac{1}{28}}_{\frac{7}{84}\left(\frac{1}{12}\right)}^{4\text{ ad }3}\quad\frac{1}{36}\ \ \frac{1}{45}\ \ \frac{1}{55}\ \ \frac{1}{66}\ \ \frac{1}{78}$$

$$\underbrace{\frac{5}{30}\left(\frac{1}{6}\right)\qquad 2\text{ ad }1\qquad\frac{7}{84}\left(\frac{1}{12}\right)}_{\frac{1}{4}}$$

$$\underbrace{\frac{1}{3}\quad+\quad\frac{1}{10}}_{\frac{13}{30}}+\underbrace{\frac{1}{21}\quad+\quad\frac{1}{36}}_{\frac{57}{21\frown 36}}+\underbrace{\frac{1}{55}\quad+\quad\frac{1}{78}}_{\frac{133}{66\frown 78}},\qquad\text{etc.}$$

auferendo 1 a numeratoribus fiet:

$$\frac{12}{30}\Big|\frac{2}{5}\qquad\frac{56}{21\frown 36}\Big|\frac{8}{3\frown 36}\Big|\frac{2}{27}\qquad\frac{12}{6\frown 78}\Big|\frac{2}{78}.\qquad\text{pendet}$$

a quadratura hyperbolae ni fallor.

3–431,10 *Nebenrechnungen*:

3+430,2+431,10 [*Z. 13–15 in der Vorlage als Spalten geschrieben*]

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120
	1	4	10	20	35	56	84	120	165	220	286				

3 $\frac{28+21\sqcap 49}{21\frown 28}\sqcap\frac{7}{84}\Big|\frac{1}{[12]}\qquad\frac{1}{6}+\frac{1}{12}\sqcap$ [*bricht ab*]

9 1
~~13~~2 ⨍ 11
~~122~~
~~1~~

9 $\frac{2}{78}$. *(1)* Unde quadratura hyperbolae *(2)* pendet *L* 16 14 *L ändert Hrsg.*

$$1\quad \underbrace{\overset{2\ \text{ad}\ 1}{\frac{1}{3}\quad\frac{1}{6}}}_{\displaystyle \frac{3}{6}\Big|\frac{1}{2}\atop A}\quad \underbrace{\overset{3\ \text{ad}\ 2}{\frac{1}{10}\quad\frac{1}{15}}}_{\displaystyle \frac{25}{150}\Big|\frac{1}{6}\atop \alpha}\quad \underbrace{\overset{4\ \text{ad}\ 3}{\frac{1}{21}\quad\frac{1}{28}}}_{\displaystyle \frac{49}{21\frown 28}\Big|\frac{1}{12}\atop a}\quad \underbrace{\overset{5\ \text{ad}\ 4}{\frac{1}{36}\quad\frac{1}{45}}}_{\displaystyle \frac{81}{36\frown 45}\Big|\frac{1}{20}}\quad \underbrace{\overset{6\ \text{ad}\ 5}{\frac{1}{55}\quad\frac{1}{66}}}_{\displaystyle \frac{121}{55\frown 66}\Big|\frac{1}{30}}\quad \underbrace{\overset{7\ \text{ad}\ 6}{\frac{1}{78}\quad\frac{1}{91}}}_{\displaystyle \frac{169}{78\frown 91}\Big|\frac{1}{42}}\quad \underbrace{\overset{8\ \text{ad}\ 7}{\frac{1}{105}\quad\frac{1}{120}}}_{\displaystyle \frac{225}{105\frown 120}\Big|\frac{1}{56}}$$

$$\underbrace{\alpha \qquad\qquad a}_{\displaystyle \frac{3}{12}\Big|\frac{1}{4}\atop B}\qquad \underbrace{\frac{81}{36\frown 45}\Big|\frac{1}{20}\qquad \frac{121}{55\frown 66}\Big|\frac{1}{30}}_{\displaystyle \frac{50}{600}\Big|\frac{1}{12}\atop \beta}\qquad \underbrace{\frac{169}{78\frown 91}\Big|\frac{1}{42}\qquad \frac{225}{105\frown 120}\Big|\frac{1}{56}}_{\displaystyle \frac{98}{42\frown 56}\Big|\frac{1}{6\frown 4}\Big|\frac{1}{24}\atop b}$$

$$\underbrace{\beta \qquad\qquad\qquad b}_{\displaystyle \frac{3}{24}\Big|\frac{1}{8}\atop C}$$

---

3 f. *Nebenrechnungen*:

$$\begin{array}{r} 13\\ 13\\ \hline 39\\ 13\\ \hline 169 \end{array}\qquad \begin{array}{l} 1\\ \not{7}8 \ ⨍\ 6\\ \not{1}3 \end{array}\qquad \begin{array}{l} 2\\ \not{9}1 \ ⨍\ 7\\ \not{1}3 \end{array}\qquad \begin{array}{r} 15\\ 7\\ \hline 105 \end{array}\qquad \begin{array}{r} 56\\ 42\\ \hline 98 \end{array}\qquad \begin{array}{l} \not{2}\\ \not{9}\not{8} \ ⨍\ 14\\ \not{7}\not{7} \end{array}$$

Patet seriem $A.\,B.\,C.$ esse geometricam, seriem $A.\,\alpha.\,a.$ etc. esse dimidiam datae, seriem $B.\,\beta.\,b.$ etc. esse dimidiam ipsius $A.\,\alpha.\,a.$

Hanc methodum, qua series fractionum triangularium abit in progressionem [geometricam] duplam invenit Hugenius. Unde deduxit fractiones triangulares aequari binario seu $\frac{2}{1}$. Ego cum aliter idem et praeterea invenissem, pyramidales aequari $\frac{3}{2}$, et triangulo-triangulares $\frac{4}{3}$ etc. et ita pyramidales summae progressionis geometricae triplae, triangulo-triangulares quadruplae, dixi ei, videri mihi similem fieri posse in aliis quoque ad geometricas reductionem quod nunc tentabo.

$$\begin{array}{ccccccccccc} [4 \text{ ad } 1 & & 2 \text{ ad } 1 & & 8 \text{ ad } 5 & & 10 \text{ ad } 7] & & & & \\ 1 & \frac{1}{4} & \frac{1}{10} & \frac{1}{20} & \frac{1}{35} & \frac{1}{56} & \frac{1}{84} & \frac{1}{120} & \frac{1}{165} & \frac{1}{220} & \frac{1}{286} \end{array}$$

$$\underbrace{\frac{1}{10}\quad\frac{1}{20}\quad\frac{1}{35}}_{\frac{[5]}{28}} + \frac{1}{56} \sqcap \left[\frac{11}{56}\right]$$

1 [*bricht ab*]

10–12 *Nebenrechnung, gestrichen*: $\frac{3}{20} + \frac{1}{35} \sqcap \frac{105+20}{} \sqcap \frac{125}{20 \frown 35}\Big|\frac{5 \frown 5}{4 \frown 7}$ [*!*]

3 qua *(1)* res *(2)* series *L* 3 f. progressionem | arithmeticam *ändert Hrsg.* | | duplam *erg.* |, | primus *gestr.* | invenit *L* 5 aliter ... praeterea *erg. L* 9 4 ... 7 *gestr. L, erg. Hrsg.* 11 25 *L ändert Hrsg.* 11 $\frac{50}{56} + \frac{1}{28} \sqcap \frac{52}{56}\Big|\frac{26}{28}\Big|\frac{13}{14}$ *L ändert Hrsg.* 13 *(1)* $\frac{5}{20} + \frac{3}{20} \sqcap \frac{8}{20}\Big|\frac{4}{5}$ *(!) (2)* $\frac{3}{20}$ *L*

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4 f. Hugenius: Zur Lösung von Huygens s. *HO* XIV S. 144–150. — Ego: vgl. N. 2 sowie die *Accessio*, *LSB* III, 1 N. 2 S. 5–9.

38_7. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS QUINTA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 12–13. 1 Bog. 2°. 4 S. Geringer Textverlust durch Randschäden.
Cc 2, Nr. 775 A tlw.

Pars V[ta] schediasmatis de seriebus et summis quadratricibus etc.

Esto $y^3 + y^2 + y + 1 + \dfrac{1}{y} \sqcap z$.

Multiplicetur per y, fiet $y^4 + y^3 + y^2 + y + 1 \sqcap yz$. facile ergo haberi potest summa omnium yz. Sed non ideo fateor summa omnium z habebitur.

Exemplo, formulam $\odot$ denuo comprobemus, resumto calculo, ponendo $h.\,l.\,c.\,m. \sqcap 0$.

Esto figura: $\dfrac{dy^3 + ey^2 + fy + g}{ny + p} \sqcap x$. fiet:

$$\left.\frac{-dy^3 - ey^2 - fy - g}{ny + p} + \frac{\begin{array}{l} dy^3 + ey^2 + fy + g \\ 3dy^2\beta + 2ey\beta + f\beta \\ 3dy\beta^2 + e\beta^2 \\ d\beta^3 \end{array}}{\begin{array}{c} ny + p \\ n\beta \end{array}}\right\} \sqcap \text{ eius differentiae } z.$$

et multiplicando per crucem:

6 $+\dfrac{1}{\mathrm{y}}$ | omniumque summa *gestr.* | $\sqcap$ z *L* 10 Esto ... $\sqcap$ x. *erg.* *L*

9 formulam $\odot$: Leibniz nimmt die Überlegungen von N. 38_5 S. 415 Z. 12 u. S. 419 Z. 4–16 wieder auf.

$$\begin{array}{llll|llll}
\boxed{-ndy^4} & \boxed{-ney^3} & \boxed{-nfy^2} & \boxed{-gny} & \multicolumn{4}{l}{\boxed{+ndy^4 \quad +ney^3 \quad +nfy^2 \quad +ngy}} \\
\multicolumn{3}{l}{\boxed{-ndy^3\beta - ney^2\beta - nfy\beta}} & -gn\beta & 2\boxed{3}ndy^3\beta & +\boxed{2}ney^2\beta & \boxed{+nfy\beta} & \boxed{+pg} \\
\multicolumn{2}{l}{\boxed{-pdy^3 \quad -pey^2}} & \boxed{-fpy} & \boxed{-gp} & 3ndy^2\beta^2 & \underline{+\ ney\beta^2} & \boxed{+pfy} & \\
 & & & & \underline{ndy\beta^3} & \boxed{+\ pey^2} & \underline{\underline{+pf\beta}} & \\
 & & & & \boxed{pdy^3} & +2pey\beta & & \\
 & & & & 3pdy^2\beta & \underline{\underline{+\ pe\beta^2}} & & \\
 & & & & 3pdy\beta^2 & & & \\
 & & & & \underline{\underline{pd\beta^3}} & & &
\end{array}$$

dividenda per $\begin{array}{l} n^2y^2 + n^2y\beta + pn\beta. \\ \quad + pny \quad + p^2 \\ \quad + pny \end{array}$ Unde fiet : $\dfrac{\begin{array}{l} 2ndy^3 + \quad ney^2 + 2epy + pf \\ \qquad\qquad 3pd.. \qquad\quad [-]ng \end{array}}{n^2y^2 + 2pny + p^2}$ ⊓ z.

Ponamus f ⊓ $\dfrac{-ep^2n - dp^3 - [n^3]g}{np}$, dividi poterit fractio per $ny + p$. Nam:

$$\begin{array}{l}
2ndy^3 + ne\ y^2 \quad + 2epy + \quad pf \\
\qquad\quad + 3pd\ .. \qquad\qquad [-]\ ng \\
\boxed{ny \quad + \ p} \qquad\qquad \boxed{ny \quad + \quad p} \\
\qquad \boxed{-\,2dp\ ..} \\
\qquad\quad \boxed{ny \quad + \quad p}
\end{array}
\quad ⨍ \quad 2dy^2 \ \dfrac{\left\{\begin{array}{l} +ney \\ +pd.. \end{array}\right.}{n} \ \dfrac{\left\{\begin{array}{l} +\boxed{2}epn \\ \quad \boxed{-enp} \\ -\ dp^2 \end{array}\right.}{n^2}$$

$$\dfrac{\left\{\begin{array}{l} -nep.. \\ -p^2d.. \end{array}\right.}{n} \ \dfrac{\left\{\begin{array}{l} -\ ep^2n \\ +\ dp^3 \end{array}\right.}{n^2}$$

10 + *L ändert Hrsg.* 14 n^2 *L ändert Hrsg.*

Unde patet, ut divisio sit exacta debere $n^2pf[-]n^3g - ep^2n + dp^3$ esse $\sqcap$ 0. et erit $f \sqcap \dfrac{ep^2n - dp^3[+]n^3g}{n^2p}$.

Recte divisus est, sed iam video productum $\dfrac{\begin{array}{c} 2dn^2y^2 + n^2ey + epn \\ \quad + npd \quad - dp^2 \end{array}}{ny \quad + p[, \frown n^2]} \sqcap z$. rursus dividi posse per $ny + p$. ideoque nullam inde aequationem ad hyperbolam oriri.

Etsi nulla series habeat differentias exhaustibiles, nisi quae sit paraboloeidum aut ex illis compositarum:

433,15–22 *Kontrollrechnung*: Proba haec est, multiplicando

$$2dy^2 \quad \dfrac{\left\{\begin{array}{l} +ne\,y \\ +pd\,.. \end{array}\right.}{n} \; \dfrac{\left\{\begin{array}{l} +epn \\ -dp^2 \end{array}\right.}{[n^2]}$$

$$\text{per} \qquad n\;y \;+\; p$$

$$\text{fiet} \qquad +2dpy^2 \; \dfrac{\left\{\begin{array}{l} +p\not{n}e\;y \\ +p^2d\;.. \end{array}\right.}{n} \; \dfrac{\left\{\begin{array}{l} +ep^2n \\ -dp^3 \end{array}\right.}{[n^2]}$$

$$\dfrac{\left\{\begin{array}{l} ep\not{n}^{\not{2}}\;.. \\ -dp^2\;.. \end{array}\right.}{n}$$

[*bricht ab*]

433,16+434,1 + *L ändert Hrsg. zweimal* 2 − *L ändert Hrsg.* 4 $, \frown$ n^2 *erg. Hrsg.* 11+15 n *L ändert Hrsg. zweimal*

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6 differentias exhaustibiles: Leibniz nimmt die Überlegung von N. 38_5 S. 421 Z. 22 – S. 422 Z. 26 wieder auf.

E				
a				
	F			
	$a-b$			
M		G		
b		$a-2b+c$		
	N		H	
	$b-c$		$a-3b+3c-d$	
		P		L
c		$b-2c+d$		$a-4b+6c-4d+e$
			Q	
	$c-d$		$b-3c+3d-e$	
d		$c-2d+e$		
	$d-e$		$c-3d+3e-f$	
e		$d-2e+f$		
	$e-f$			
f				

Videndum est an ipsarum differentiarum transversalium series possint exhaustibiles esse, ut $E.\ F.\ G.\ H.$ et patet eam seriem non posse esse arithmeticam, foret enim $E-F$, seu $\boxed{a-a}+b \sqcap F-G$. seu $a-b$, $[-a+2b-c]$. seu foret $b \sqcap b[-]c$. quod est absurdum.

Videamus an possit esse quadratica; tunc erit

$$\llcorner E-F \lrcorner - \llcorner F-G \lrcorner \sqcap \llcorner G-H \lrcorner \quad - \llcorner H-L \lrcorner .$$
$$b \qquad b[-]c \qquad b+d-2c \qquad b-3c+3d-e$$

Sed hoc duo aequari absurdum est, et demonstratio generalis in promtu est, cur sit impossibile ullam seriem differentiarum transversalium exhauriri posse, posito, seriem $a.\,b.\,c.\,d.$ in infinitum extendi, quia seriei $E.\ F.\ G.\ H.\ L.$ differentiae sunt $M.\ N.\ P.\ Q.$ etc. et huius rursus, seriei sequentis termini, et ita in infinitum.

In progressione harmonica series $a.\ b.\ c.\ d.\ e.$ etc. eadem seriei $F.\ G.\ H.\ L.$ et per consequens etiam series $M.\ N.\ P.\ Q.$ seriei $b-c.\ \ c-d.\ \ d-e.$ etc.

434,7–435,18 compositarum: ... *(1)* Differentiarum autem transversalium ut EFGH seriem impossi *(2)* Videndum *L* 20 a + 2b + c *L ändert Hrsg.* 20+23 + *L ändert Hrsg. zweimal*

$$1-\frac{1}{1} \quad 2-\frac{1}{2} \quad 3-\frac{1}{3} \quad 4-\frac{1}{4} \quad \text{dant:} \quad 0 \quad +\frac{3}{2} \quad +\frac{8}{3} \quad +\frac{15}{4} \quad +\frac{24}{5}$$

$$\frac{3}{2} \quad \frac{7}{6} \quad \frac{13}{12} \quad \frac{21}{20} \quad \text{etc.}$$

Sed talis series componitur ex duabus summabilibus: fractionum triangularium, et potestatum.

Si a serie quadam auferatur eius series summatrix sed uno gradu inferior, ut

$b, -a+b \qquad c, -b+c \qquad d, -c+d$ fiet:

inde $2b-a \qquad 2c-b \qquad 2d-c$ fiet series composita ex data $b.\ c.\ d$; et differentiis $b-a \quad c-b \quad d-c$ etc.

Idem est si in unum addas:

$$a\boxed{-b,b}-c \qquad \underbrace{a-2b+c,b-2c+d}_{a-b-c+d} \qquad \underbrace{a-3b+3c-d,+b-3c+3d-e}_{a-2b*c+2d-e} \qquad \text{etc.}$$

quorum differentiae:

$a-c \qquad -b+d \qquad -b+c+d-e$

[*Hier folgt das Schema auf der gegenüberliegenden Seite.*]

$$\underset{A}{\frac{1}{y^2}} - \frac{1}{y^2+2y+1} \sqcap \underset{B}{\frac{\boxed{y^2}+2y+1\boxed{-y^2}}{y+1,\frown y,,\square}} \text{ quorum habetur summa. Ergo } \frac{2y}{y+1,\frown y,,\square} \sqcap$$

$$\frac{2}{y+1,\square,\frown y} \text{ et } \frac{1}{y+1,\frown y,,\square} \text{ pendent alterum ex altero.}$$

Data serie quadam videndum an ei addi vel adimi possit alia series, ita ut post ablationem vel additionem producta series sit eadem sed uno tantum gradu superior vel inferior additae[,] ademtae, tunc enim ea series erit propositae quadratrix.

8 differentiis *erg. L* 15 f. altero. *(1)* Addatur ad *(2)* Ad $\frac{2y+1}{y+1,y,,\square}$ addatur $\frac{y,\square}{y+1,\frown y,,\square} \sqcap \frac{1}{y+1,\square}$, fiet: $\frac{1}{y^2}$ *(3)* Data *L* 18 additae *erg. L*

11 $*\ c$: c hat den Koeffizienten 0.

$\frac{1}{1}$		
	$\frac{3}{4}$	
$\frac{1}{4}$		$\frac{88}{144}$
	$\frac{5}{36}$	
$\frac{1}{9}$		
	$\frac{7}{9 \frown 16}$	
$\frac{1}{16}$		
	$\frac{9}{16 \frown 25}$	
$\frac{1}{25}$		
	$\frac{11}{25 \frown 36}$	
$\frac{1}{36}$		
	$\frac{13}{36 \frown 49}$	
$\frac{1}{49}$		
	$\frac{15}{49 \frown 64}$	
$\frac{1}{64}$		
	$\frac{17}{64 \frown 81}$	
$\frac{1}{81}$		
	$\frac{19}{81 \frown 100}$	
$\frac{1}{100}$		

Ad $\frac{2y+1}{y+1,\frown y,,\square}$ addatur $\frac{1}{y+1,\frown y,,\square}$. $\frac{2y+2}{y+1,\frown y,,\square}$, fiet: $\frac{2}{y+1,\frown y^2}$, sive $\frac{2}{y+1,\frown y,\frown y}$.

Ita tres ergo series: $\frac{1}{y+1,\square,\frown y}$. $\frac{1}{y+1,y,,\square}$. $\frac{1}{y+1,y^2}$ pendent a se invicem, ita ut una inventa et reliquae habeantur. Iam secundae momentum ex y. dat primam; eiusdem secundae momentum ex $y+1$. dat tertiam. Tertia ergo differt a [prima] cylindro mediae, quod et per se patet, quia et ordinatae extremarum differunt ⟨ordinata mediae⟩.

Fiat series, ipsarum p, ita ut $\frac{p}{w[\sqcap] \text{ diff. ord.}} \sqcap \frac{\text{ord.}}{1}$, fiet $p \sqcap \frac{\text{ord.} \frown \text{diff. ord.}}{1}$. Cui si addantur triangula semiquadrata [differentiarum] ordinatarum, habebitur series summabilis. Unde: si ordinatae $\frac{1}{y}$. differentiae ordinatarum $\frac{1}{y^2+y}$. fiet $p \sqcap \frac{1}{y^3+y^2} \sqcap \frac{1}{y+1,\frown y^2}$. addatur $\frac{1}{y+1,y,\square,2}$. summa huius seriei erit [semiquadratum]. Itaque se-

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3 *Über* $\frac{1}{y+1,y,,\square}$, *mit Hinweisstrich verbunden*: Haec series media est fractionum triangularium quadratarum.

10 *Über* addatur: Imo subtrahi debet, haec causa erroris porrecti usque ad signum ☽.

436,18–438,1 quadratrix. *(1)* A serie *(2)* Differentia differentiae: $\frac{2\text{y}+1}{\text{y}+1,\frown \text{y},,\square}-$ *(3)* Ad *L* 4 Iam *(1)* primae *(2)* secundae *L* 5 f. a | prima *ändert Hrsg.* | cylindro mediae *(1)*. Ergo summa seriei $\frac{1}{\text{y}+1,\frown \text{y}^2} - \frac{1}{\text{y}+1,\square,\frown \text{y}}$ seu $\frac{\text{y}+1-\text{y}}{\text{y}+1,\text{cub},\text{y}^4}$ *(2)*, quod *L* 7 $\sqcap$ *erg. Hrsg.* 7 f. $\frac{\text{ord.} \frown \text{diff. ord.}}{1}$ *(1)* , horum summa habetur triangulo semiquadrato ordinatae maximae *(2)*. Cui *L* 8 differentiarum *erg. Hrsg.* 8 habebitur *(1)* figura *(2)* series *L* 10 quadratum *L ändert Hrsg.*

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9 si ordinatae $\frac{1}{y}$: Leibniz nimmt die Überlegung von 38₄ S. 404 Z. 5–8 wieder auf. 11 f. fractionum triangularium quadratarum: Im Nenner stehen die Quadrate der verdoppelten Dreieckszahlen; der Fehler tritt in S. 439 Z. 13, in N. 38₈ S. 445 Z. 14 und in N. 38₉ S. 460 Z. 10 f. wieder auf.

ries $\frac{1}{y^3+y^2}$, etiam [ex] quadratis triangularium pendet. Habemus ergo quatuor series, quarum una habita, caeterae omnes habentur, nempe:

$$\underset{A}{\frac{1}{y+1,\square,y}} \qquad \underset{B}{\frac{1}{y+1,y,\square}} \qquad \underset{C}{\frac{1}{y+1,y^2}}.$$

$C-A \sqcap B$. $\frac{B}{2}+C \sqcap \underline{cognito}$. Ergo $2C-A \sqcap \underline{cognito}$.

Iam $2A+B$ aequatur $\underline{\underline{cognito}}$ per superiora.

Ergo $C-A \sqcap \underline{\underline{cognito}}-2A$. Ergo $C \sqcap \underline{\underline{cognito}}-A$.

At idem A aequatur $2C-\underline{cognito}$. Ergo $C \sqcap \underline{\underline{cognito}}-2C+\underline{cognito}$.

Ergo $\boxed{3}C \sqcap \frac{\underline{\underline{cognito}}+\underline{cognito}}{3}$. Unde sequitur has tres series repertas.

Data serie summatrice, seriei C, necesse est et seriei summatricis eius dari summam, quia, momentum seriei C. ex y. datur.

Data summa seriei C, dabitur et summa omnium $\frac{1}{y^2}$, quod ita ostendo, momentum seriei C seu $\frac{1}{y+1,y^2}$ ex y, habetur; est enim summa fractionum triangularium. Momentum autem seriei C, ex $y+1$. est $\frac{1}{y^2}$. quod a priore differt cylindro seriei C.

Operae ergo pretium est, ut ista nonnihil resumamus.

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14 *Am Rande*: ☽

1 ex *erg. Hrsg.* 1 ergo *(1)* quinque *(2)* quatuor *L* 9 f. repertas. *(1)* Reperta serie C *(2)* Data *L* 14 cylindro *(1)* figurae *(2)* seriei *L*

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5 $2C-A \sqcap \underline{cognito}$: Richtig wäre $3C-A=2\,\underline{cognito}$. Leibniz rechnet konsequent weiter. Die Überlegung wird dadurch bis Z. 9 zusätzlich beeinträchtigt. 13 fractionum triangularium: s. o. Erl. zu S. 438 Z. 11 f.

$\frac{1}{y^2} - \frac{1}{y^2+2y+1} \sqcap\!\!\!\sqcap$ *cognitis* $d \sqcap \frac{1}{b^2}$. Ergo $\frac{\boxed{y^2}+2y+1\boxed{-y^2}}{y+1,y,\square} \sqcap\!\!\!\sqcap \frac{1}{b^2}$. ponendo b esse minimam abscissarum, y[,] et $\frac{1}{b^2}$ maximam ordinatarum, in nostro casu.

Ergo $\underset{2A}{\frac{2}{y+1,\square,y}} + \underset{B}{\frac{1}{y+1,y,\square}} \sqcap\!\!\!\sqcap \frac{1}{b^2}$. Ad $\underset{2A+B \sqcap\!\!\!\sqcap \frac{1}{b^2}}{\frac{2y+1}{y+1,y,\square}}$ addatur $\underset{B}{\frac{1}{y+1,y,\square}}$ fiet:

$2A+2B \sqcap \frac{2y+2}{y+1,y,\square} \sqcap\!\!\!\sqcap \frac{1}{b^2} + B \sqcap\!\!\!\sqcap \frac{2}{y+1,y^2} \sqcap 2C.$

Ergo $\frac{1}{b^2} \sqcap\!\!\!\sqcap 2C - B \sqcap 2A+B$. Ergo vel una ex his tribus seriebus separatim data caeterae habentur, unde rursus patet, vel unica ex his seriebus data haberi ipsam $\frac{1}{y^2}$. Sed nunc pergamus:

Sumatur iam series harmonica $\frac{1}{y}$, differentia ordinatarum $\frac{1}{y} - \frac{1}{y+1} \sqcap \frac{\boxed{y}+1\boxed{-y}}{y^2+y} \sqcap \frac{1}{y+1,y}$. Fiat terminus seriei cuiusdam alterius qui sit, $\underset{C}{\frac{1}{y} \frown \frac{1}{y+1,y}}$: auferatur ab eo semiquadratum differentiae, seu $\frac{1}{y+1,y,\square,2} \sqcap \frac{B}{2}$. summa erit $\frac{1}{2b^2}$ semiquadratum ab $\frac{1}{b}$ ordinata maxima. Erit $C - \frac{B}{2} \sqcap\!\!\!\sqcap \frac{1}{2b^2}$. seu $2C - B \sqcap\!\!\!\sqcap \frac{1}{b^2}$, ut ante. Nondum ergo ipsa vel C, vel B, vel A, habetur. Superest ut ostendam ex data C, et per consequens aliqua alia, quas dixi dari $\frac{1}{y^2}$.

1 f. ponendo ... casu. *erg.* *L* 9 series *(1)* $\frac{1}{\mathrm{y}^2}$, differentia ordinatarum *(a)* $\frac{1}{\mathrm{y}^2+2\mathrm{y}+1}$ *(b)* $\frac{2}{\mathrm{y}}$ *(c)* $\frac{1}{\mathrm{y}^2} - \frac{1}{\mathrm{y}^2+2\mathrm{y}+1} \sqcap \mathrm{y}^2+2\mathrm{y}+1-\mathrm{y}^2$ *(2)* harmonica *L* 12 f. semiquadratum ... maxima *erg.* *L*

Nam momentum ipsius C, ex y, est $\frac{1}{y^2+y}$ quod est $\sqcap\!\sqcap\ \frac{1}{b}$. Momentum ipsius C, ex $y+1$, est $\frac{1}{y^2}$. Ergo differentia inter $\frac{1}{y^2}$, et $\frac{1}{y^2+y}$ erit C. Nam $\frac{y^2+y-y^2}{y^2 \frown y^2+y} \sqcap \frac{1}{y+1, y^2}$.

Ergo $\frac{1}{y^2} - \frac{1}{b} \sqcap\!\sqcap C$. Ergo $\frac{1}{by^2} - \frac{1}{b^2} \sqcap\!\sqcap \left[\frac{C}{b}\right] \sqcap\!\sqcap \frac{1}{by^2} + B - 2C$. Unde $C \sqcap\!\sqcap \dfrac{\frac{1}{by^2}+B}{\left[\frac{1}{b}\right]+2}$.

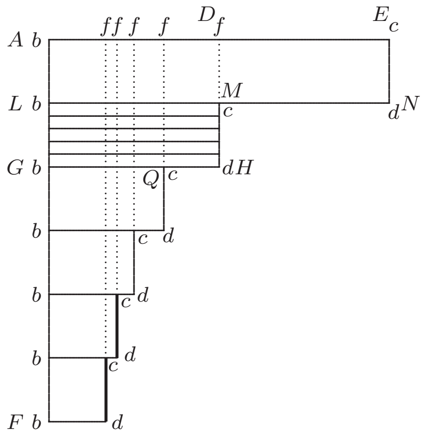
Aliquando ex hac aequationum inter summas combinatione aliquid elici poterit.

Series autem $\frac{1}{2y^2}$ est momentum seriei $\frac{1}{y}$ ex axe. Idem momentum vero et aliter per regulam superiorem haberi potest, (vide part. 2. huius schediasmatis et ibi fig. pag. eius plagulae 2.). Nimirum rectangula ut MNE vel QHP sub abscissa figuram complent. Ergo

4 *Am Rande*: Adde infra schediasm. part. VII. pag. eius plagulae 3. sub fin.

3 Cb *L ändert Hrsg.* 3 b *L ändert Hrsg.* 7 abscissa *(1)* constituunt figuram, ergo *(2)* figuram *L*

6 vide: N. 38₃ Fig. 2:



8 infra: s. u. N. 38₉ S. 461 Z. 2–5.

et figurae momentum componitur ex rectangulorum momentis, rectanguli autem cuiusque momentum fit ex ductu ipsius rectanguli in distantiam eius centri gravitatis, ab axe AG. Nimirum ordinata AE vel LN fit rectangulum $ALEN$ esto $\frac{1}{1}$, abscissa eius 1. AL. Differentia huius ordinatae a sequenti, est rectangulum MNE, sed si n u l l a sequatur, in computo, seu si is hic terminetur, tunc ipsa ordinata censebitur eius differentia ab ultima 0 seu zero.

Semper ergo momentum figurae v. g. [$AGHMNE$] ita habebitur[,] differentia ordinatarum, quae hoc loco unica est $MN[\sqcap]\frac{1}{2}$ ducatur in abscissas hoc loco in $AL, \sqcap 1$ fient rectangula ut hoc loco $MNE \sqcap 1 \frown \frac{1}{2} \sqcap \frac{1}{2}$. ducatur quodlibet rectangulum in ordinatam suam hoc loco MNE in $LN \sqcap MN$. sed dimidia PE, minutam, seu $LN - \frac{PE}{2} \sqcap \frac{3}{4}$ hoc loco fiet $\frac{3}{8}$. Summae omnium horum solidorum addatur momentum ipsius GHP scilicet ultima ordinatarum GH, hoc loco $\sqcap \frac{1}{2}$ ducta in suam abscissam 2, fiet rectangulum $GHP \sqcap 1$. cuius momentum fit si ducatur in $\frac{GH}{2}$ dimidiam ordinatam, $\sqcap \frac{1}{4}$ erit eius momentum, $\frac{1}{4}$. Addatur $\frac{3}{8}+\frac{2}{8}$ fiet $\frac{5}{8}$, sive $\frac{1}{2}+\frac{1}{8}$ quod est figurae momentum. Idem autem provenit, si sumas semiquadrata ordinatarum nempe $\frac{1\frown 1}{2}, +\frac{\frac{1}{2}\frown\frac{1}{2}}{2}$ seu $\frac{1}{2}+\frac{1}{[8]}$. Sic ergo generaliter:

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15 *Am Rande*: $\frac{1}{2}+\frac{1}{8}$

2 ipsius *erg. L* 3 Nimirum *(1)* abscissa AE *(2)* ordinata AE vel LN | *(a)* seu rectangulum *(b)* fit rectangulum ALEN *erg.* | esto *L* 4 ordinatae *(1)* a praecedente, est ipsamet ordinata *(2)* a *L* 7 AHGHMNE *L ändert Hrsg.* 8 loco *(1)* unica *(2)* duae *(3)* unica est. *(a)* Rectangulum MNE *(b)* MN | $\sqcap$ *erg. Hrsg.* | $\frac{1}{2}$ *(aa)* ductis, in *(bb)* et GH, ordinata ultima, $\frac{1}{2}$ *(aaa)* ductis in *(bbb)* ducantur in abscissas, prior in AL $\sqcap$ 1, posterior in AG $\sqcap$ 2, fiunt rectangula MNE $\sqcap$ $\frac{1}{2}$, et GHP $\sqcap$ 1 *(c)* ducatur *L* 8 f. AL, $\sqcap$ 1 *(1)* eorum summa hoc loco *(a)* triangula *(b)* fient *(2)* fient rectangula | ut hoc loco *erg.* | MNE *L* 10 sed *(1)* abscissa *(2)* dimidia *L* 11 momentum ... scilicet *erg. L* 12 hoc loco *erg. L* 15 4 *L ändert Hrsg.*

$\frac{x^2}{2} \sqcap\!\!\!\sqcap ywx - \frac{yw^2}{2} + \frac{e^2b}{2}$, ponendo y abscissam, x ordinatam, w differentiam [ordinatarum], e ultimam ordinatam[,] b ultimam abscissam. Quae est reg. [6.] schediasm. part. 2.

Unde duci potest corollarium semper haberi summam seriei $\frac{x^2 + yw^2 - 2ywx}{\not 2} \sqcap \frac{e^2b}{\not 2}$. Quod ut exemplo nostro applicemus fiet $\frac{1}{y^2} + \frac{1}{y+1, \square, y} - \frac{2}{y^2+y} \sqcap\!\!\!\sqcap e^2b \sqcap \frac{1}{b}$. Iam $\frac{2}{y^2+y} \sqcap\!\!\!\sqcap \frac{2}{b}$. Ergo (1) $\frac{1}{y^2} + \underset{A}{\frac{1}{y+1, \square, y}} \sqcap\!\!\!\sqcap e^2b + \frac{2}{b}$. Iungamus duas aequationes supra inventas: (2) $\frac{1}{b^2} \sqcap\!\!\!\sqcap 2C - B \sqcap 2A + B$ (3). ☿ Ergo (4) $C \sqcap A + B$ et (5) $\frac{1}{y^2} - \frac{1}{b} \sqcap C$. Ergo (6) $\frac{1}{y^2} - \frac{1}{b} \sqcap A + B$ per 5. et 4. Iam $B \sqcap \frac{1}{b^2} - 2A$. per 2. et 3. Ergo $\frac{1}{y^2} - \frac{1}{b} \sqcap\!\!\!\sqcap \boxed{A} + \frac{1}{b^2} - \boxed{2}A$. Iam $-A \sqcap \frac{1}{y^2} - e^2b + \frac{2}{b}$ per aeq. 1. et fiet: $\boxed{\frac{1}{y^2}} - \frac{1}{b} \sqcap \frac{1}{b^2} \boxed{+\frac{1}{y^2}} - e^2b + \frac{2}{b}$.

Error calculi in eo quod scilicet ordinatam primam quae differentiarum summa est, cum ultima, confudi. Aequatio, in qua ultima ordinata adhibetur ut ubi est e^2b servit tantum ad finite productarum serierum inveniendas summas.

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8 *Zu* ☿ *am Rande*:

$$A \quad \frac{1}{y+1, \square, y}$$

$$B \quad \frac{1}{y+1, y, \square}$$

$$C \quad \frac{1}{y+1, y^2}$$

1 Quae est reg. | 2. *ändert Hrsg.* | schediasm. part. 2. *erg. L* 1 f. abscissarum *L ändert Hrsg.* 10 per aeq. 1. *erg. L* 12 ut | ut *streicht Hrsg.* | ubi *L*

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2 reg. [6.]: s. o. N. 38₃ S. 399 Z. 1–3.

Ergo appellando ordinatas inconstantes x. abscissas inconstantes y. abscissam b. maximae ordinatae, e. abscissam d. minimae ordinatae, h. fient regulae:

$\frac{x^2}{2} \mathrel{\sqcap\!\!\!\!|}\; ywx - \frac{yw^2}{2} + \frac{d^2h}{2}$. Et pari iure $\frac{y^2w}{2} + \frac{d^2h}{2} \mathrel{\sqcap\!\!\!\!|}\; xy - \frac{x}{2}$. $e - h \mathrel{\sqcap\!\!\!\!|}\; w$. $xw \mathrel{\sqcap\!\!\!\!|}\; \frac{e^2}{2} - \frac{w^2}{2}$. $yw \sqcap x$. in decrescentibus. Nam in ascendentibus, seu crescentibus $yw \mathrel{\sqcap\!\!\!\!|}\; eb - x$.

Variandae hae regulae nonnihil prout series crescunt aut decrescunt, item omitti poterat mentio minimae ordinatae; intelligendo eam semper esse ultimam w. Contra ubique inseri etiam potest, ubi ipsarum w, mentio est.

Omnes series hactenus inventae per unam ex his regulis habentur; exceptis seriebus potestatum, quae habentur per differentias exhaustas. Excutiendum adhuc hoc argumentum, quando in numeris mutare licet abscissam in ordinatam et contra. Ita $x \sqcap \frac{1}{y}$. Unde $y \sqcap \frac{1}{x}$. Harmonica series ab utroque latere similis, ut si semper procederet ista inversio, posset haberi summa irrationalium.

$\sqrt{y+1} - \sqrt{y} \sqcap z$. Unde: $\boxed{y} + 1 \boxed{-y} - 2\sqrt{y^2+1} \sqcap z^2$. Ergo $z^4 - 2z^2 \boxed{+1} \sqcap 4y^2 \boxed{+1}$. et fiet $z\sqrt{z^2-2} \sqcap 2y$. Pro z. pone $v + \frac{1}{2}$. fiet $\sqrt{z^2-2} \sqcap \sqrt{v^2+1+\frac{1}{4}-2}$. sive $\sqrt{v^2-1+\frac{1}{4}} \sqcap v - \frac{1}{2}$, et fiet: $v^2 - \frac{1}{4} \sqcap 2y$. eritque $v \sqcap \sqrt{y+\frac{1}{4}}$. Sumta ergo serie numerorum irrationalium $\sqrt{9}\ \sqrt{8}\ \sqrt{7}\ \sqrt{6}\ \sqrt{5}\ \sqrt{4}\ \sqrt{3}\ \sqrt{2}\ \sqrt{1}$. et alia $\sqrt{8+\frac{1}{4}}\ \sqrt{7+\frac{1}{4}}\ \sqrt{6+\frac{1}{4}}\ \sqrt{5+\frac{1}{4}}\ \sqrt{4+\frac{1}{4}}\ \sqrt{3+\frac{1}{4}}\ \sqrt{2+\frac{1}{4}}\ \sqrt{1+\frac{1}{4}}$. summa erit $2.\ (\sqrt{3} - \sqrt{1}) - \frac{1}{2} \sqcap \frac{3}{2}$ [*bricht ab*]

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3 *Über der ersten Gleichung*: Reg. 6. plagulae 2^{dae}.

13 *Darüber*: Error calculi

13–17 Der Abschnitt weist eine Reihe von Rechenfehlern auf. Leibniz ändert den Ansatz zweimal durch Überschreiben in $\sqrt{y-1} - \sqrt{y}$ bzw. $\sqrt{y-1} + \sqrt{y}$, erkennt aber, daß dies nicht zum Erfolg führt. Er vermerkt die Fehlerhaftigkeit und beginnt im folgenden Teilstück von neuem. 18 Reg. 6.: s. o. N. 38_3 S. 399 Z. 1–3.

38_8. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS SEXTA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 14–15. 1 Bog. 2°. 4 S. tlw. zweispaltig. Cc 2, Nr. 775 A tlw.

Schediasmatis de serierum summis pars VI[ta].

Cogitandum est de summis irrationalium ineundis: $\sqrt{y+1} - \sqrt{y} \sqcap z$. fiet: $y+1, + \, y - 2\sqrt{y^2+y} \sqcap z^2$. sive $2y+1-z^2 \sqcap 2\sqrt{y^2+y}$. et rursus quadrando:

$\boxed{4y^2} + 4[y] - 4yz^2 + 1 - 2z^2 + z^4 \sqcap \boxed{4y^2} + 4y$. et fiet:

$z^4 \begin{array}{l} -\,2z^2 \\ -\,4y.. \end{array} \sqcap [-1]$. sive $z \sqcap \sqrt{2y+1-2\sqrt{y^2+y}}$.

Huius seriei finitae, quoniam iniri potest summa, poterit etiam summa inveniri serierum aliarum finitarum, quae constant summis differentiisque aliarum serierum summabilium cum hac.

Sed: antea videamus an haec series reddi possit simpliciter, assumta tali valore ipsarum y, ut ex $\sqrt{y^2+y}$ extrahi possit radix seu sumtis illis tantum numeris triangularibus, qui sunt quadrati. Qualis est 36.

$\boxed{y^2} + y$. aequandus quadrato $\boxed{y^2} \mp 2ym + m^2$. fiet $y \pm 2ym \sqcap m^2$. et $y \sqcap \dfrac{m^2}{1 \pm 2m}$, sive ut denominator sit affirmativus et m integer, $\dfrac{m^2}{1+2m}$ et dividendo

15 *Daneben, in der Vorlage als Spalten geschrieben:*

1	2	3	4	5	6	7	8	9
1	3	6	10	15	21	28	36	

4 pars *(1)* 5[ta] *(2)* VI[ta] *L* 7 y *erg. Hrsg.* 8 4y − 5 *L ändert Hrsg.* 10 finitae *erg. L*
16 $+m^2$. *(1)* erit $3y \sqcap m^2$ et $y \sqcap \dfrac{m^2}{3}$. Sumantur ergo omnes *(2)* fiet *L*

14 numeris triangularibus: Leibniz führt die Überlegung irrtümlich für die verdoppelten Dreieckszahlen y^2+y durch, wie er in der Anmerkung zu S. 446 Z. 4–11 erkennt, korrigiert den Text aber nicht.

$$\begin{array}{llll} & -\dfrac{m}{2} & & \\ m^2 & * & * & \\ \hline \boxed{2m\,+\,1} & & & \smallint \dfrac{m}{2} - \dfrac{1}{4} \\ & \boxed{2m+1} & & \end{array}$$

$\dfrac{m^2}{1 \ddagger 2m} \sqcap p.$ integro. Ergo $\dfrac{m^2}{p} \sqcap 1 \ddagger 2m$. Ponamus $m \sqcap pq$. fiet: $\dfrac{p^2q^2}{p} \sqcap 1 + 2pq$. Ergo $q^2p \sqcap 1 + 2pq$. $q^2p - 2pq \sqcap 1$. Unde $qp - 2p \sqcap \dfrac{1}{q}$ integer fracto, quod est impossibile. Eodem modo: $m^2 \sqcap p \ddagger 2mp$. Ergo $\dfrac{m^2}{p} \sqcap 1 \ddagger 2m$. Ergo m^2, divisibilis per p. ita tamen ut radix eius non sit divisibilis per p. Deinde $m^2 \dagger 2mp \sqcap p$, erit $m \dagger$ [2]$p \sqcap \dfrac{p}{m}$. Ergo p divisibilis per m. Ergo resumta aequatione pro p ponendo mn, fiet: $\dfrac{m^2}{1 \ddagger 2m} \sqcap mn$. Ergo $\dfrac{m}{1 \ddagger 2m} \sqcap n$. Unde: $m \sqcap n \ddagger 2mn$, et $m \dagger 2mn \sqcap n$. Ergo $1 \dagger 2n \sqcap \dfrac{n}{m}$. Ergo n, divisibilis per m. Ponamus ergo $n \sqcap mh$, fiet resumta aequatione $\dfrac{\not m}{1 \ddagger 2m} \sqcap \not m h$. et fiet $\dfrac{1}{1 \ddagger 2m} \sqcap h$. Ergo si m sit integer h, necessario est fractus.

4–447,12 Valde notabilia hic dicta sunt de integris investigandis per duplicem methodum.

4–11 Ex ratiocinatione ista patet impossibile esse ut numerus triangularis duplicatus fiat unquam quadratus.

5 +2pq. *(1)* | Sed *streicht Hrsg.* | non ita exitus an ita: *(2)* $q^2p - 2pq$ *L* 7 2 *erg. Hrsg.*

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4 $\dfrac{m^2}{1 \ddagger 2m}$: Leibniz hat die Rechnung bis Z. 5 zunächst mit dem Ansatz $\dfrac{m^2}{1+2m}$ durchgeführt und nachträglich nicht durchgehend verallgemeinert.

$$\begin{array}{l}
\qquad\quad 1 \ \pm\ 2m \frown sm + r \sqcap m^2. \\
\qquad\quad \underline{sm \ +\ \ r\quad} \\
\qquad\quad r \ \pm\ 2mr \\
\underline{\ \ sm \ \ \pm\ 2sm^2 \qquad\qquad} \\
\pm\, 2sm^2 \pm 2mr \ +\ \ r \sqcap m^2 \ \text{ sive } \pm\ 2sm^2 \pm 2rm + r \sqcap m^2. \\
\qquad + \ sm \qquad\qquad\qquad\qquad\qquad + \ s\,..
\end{array}$$

NB. ista dividendi methodus in numeris non ita procedit, ut in incognitis etiamsi ipsi numeri debeant esse incogniti seu generales ut hoc loco m, ponendo $\pm s \sqcap \frac{1}{2}$ fiet: $\boxed{m^2} \pm 2rm + r \sqcap \boxed{m^2}$. et $m \sqcap \dfrac{-r}{\pm 2r \pm \frac{1}{2}}$ sive $m \sqcap \dfrac{-2r}{\pm 4r \pm 1} \sqcap \dfrac{2r}{\mp 4r \mp 1}$. Invertendo
$\pm \frac{1}{2}$ ..

$\dfrac{\mp 4mr \mp m}{2r}$ [$\sqcap$] $1 \sqcap \mp 2m \mp \dfrac{m}{2r}$. tantum ergo opus est sumi m, multiplum ipsius $2r$. et poterit m^2, dividi per $1 \pm 2m$. id est fiet: $1 \sqcap \mp 4rt \mp t$. et erit $\dfrac{1}{\mp 4r \mp 1} \sqcap t$. seu $1 \sqcap \mp 4r \mp 1$. Ergo $r \sqcap 0$. Si r debet esse integer esse debet $m \sqcap 0$.

Ut ergo redeamus ad scopum, quia summa haberi potest huius seriei, $z \sqcap \sqrt{2y + 1 - 2\sqrt{y^2 + y}}$ et ponendo $y \sqcap \dfrac{m^2}{1 \pm 2m}$ fit $\sqrt{y^2 + y} \sqcap y \mp m$. seu $\dfrac{m^2}{1 \pm 2m} \mp m$. fiet $z \sqcap \sqrt{\boxed{2}y + 1\boxed{-y}} \pm m$ sive $z \sqcap \sqrt{\dfrac{m^2}{1 \pm 2m} + 1} \pm m$ sive $\sqrt{\dfrac{2m^2 + 1 \pm 2m\boxed{+m^2}}{1 \pm 2m}}$, sive

1–12 Notabilis haec methodus habendi numeros integros, in exemplum, etsi alioquin hic exitum non dederit.

14 *Neben* ponendo: Error ab hoc signo ⊙ usque ad sequens ☽.

5+7 +r $\sqcap$ m^2 *(1)* et ponendo s $\sqcap$ n + 1. fiet: 2nm^2 *(2)* NB. *L*   10 $\sqcap$ *erg. Hrsg.*

14 ponendo $y \sqcap \dfrac{m^2}{1 \pm 2m}$: Der folgende Ansatz, bei dem m die natürlichen Zahlen durchläuft, ist verfehlt; Leibniz erkennt dies ab S. 451 Z. 12 und markiert den ungültigen Abschnitt.   15 fiet z: Leibniz vergißt bei der Ersetzung von $2\sqrt{y^2 + y}$ den Faktor 2 und im Zähler des dritten Wurzelausdrucks den Term $\pm\, m$. Die Rechnung wird dadurch bis S. 448 Z. 9 beeinträchtigt, Leibniz setzt anschließend neu an.

$z \sqcap \dfrac{\sqrt{2m^2 \ddagger 2m + 1}}{1 \ddagger 2m}$. Sumtis ergo m numeris naturalibus; summa quotcunque irrationalium huius naturae haberi potest. Mutando signum in $+$, fiet $z \sqcap \sqrt{\dfrac{2m^2 + 2m + 1}{2m + 1}}$, sive $z \sqcap \sqrt{\dfrac{2m^2}{2m+1} + 1}$.

Ergo $\sqrt{\dfrac{2}{2+1} + 1}$ $\sqrt{\dfrac{8}{4+1} + 1}$ $\sqrt{\dfrac{18}{6+1} + 1}$ $\sqrt{\dfrac{32}{8+1} + 1}$ summari possunt. Eorum enim summa est differentia inter maximam seu potius proxime maiorem et minimam $\sqrt{y}$. Maxima est exclusa, minima inclusa.

Iam $y \sqcap \dfrac{m^2}{1 \ddagger 2m}$ seu $\dfrac{m^2}{1+2m}$. fiet $\sqrt{y} \sqcap \dfrac{m}{\sqrt{1+2m}}$. Proxime maior ergo y est $\dfrac{25}{1+10} \sqcap \dfrac{25}{11}$, cuius radix $\dfrac{5}{\sqrt{11}}$. minima autem est $\dfrac{1}{3}$, seu $\dfrac{1}{\sqrt{3}}$. Ergo $\dfrac{5}{\sqrt{11}} - \dfrac{5}{\sqrt{3}}$ aequantur summae horum quatuor irrationalium.

Res ob momentum sui resumenda est:

$$\frac{m+1}{\sqrt{\underset{3}{\boxed{1}} + 2m \boxed{+2}}} - \frac{m}{\sqrt{1+2m}} \sqcap z.$$

Ergo $\dfrac{m^2+2m+1}{\underset{3}{\boxed{1}} + 2m \boxed{+2}} + \dfrac{m^2}{1+2m} - 2 \frown \dfrac{m^2+m}{\sqrt{\begin{matrix} 3 + 2m \\ \dfrac{+2m + 4m^2}{3 + 4m + 4m^2}\end{matrix}}} \sqcap z^2$.

Ergo $\dfrac{m^2+2m+1}{3+2m} + \dfrac{m^2}{1+2m} - z^2 \sqcap \dfrac{+2m^2+2m}{\sqrt{3+4m+4m^2}}$ in quo patet esse errorem.

Resumatur res ab ovo: $\sqrt{y+1} - \sqrt{y} = z$.

1 numeris *(1)* arithmet *(2)* naturalibus *L* 5 enim *(1)* summa est: $\sqrt{\mathrm{y}+1} - \sqrt{\mathrm{y}}$. *(a)* ita mut *(b)*. Est autem *(c)* si *(2)* summa *L* 5 seu ... maiorem *erg.* *L* 6 $\sqrt{\mathrm{y}}$. *(1)* Est autem maxima ex *(2)* Maxima est *(a)* inclusa *(b)* exclusa *L*

13 $+2m+4m^2$: Richtig wäre $+6m+4m^2$. Leibniz bemerkt die Unstimmigkeit in der nächsten Zeile und setzt neu an.

Ergo $y+1+y-2\sqrt{y^2+y} \sqcap z^2$. Ergo $z \sqcap \sqrt{2y+1-2\sqrt{y^2+y}}$. Pone $y^2+y \sqcap y^2-2ym+m^2$, fiet: $y \sqcap \frac{m^2}{1+2m}$. Quare resumendo fiet: $\sqrt{\frac{m^2}{1+2m}+1-}\ \sqrt{\frac{m^2}{1+2m}} \sqcap z$. Ita NB. cum abscissae sint y, erunt quidem progressiones arithmeticae, nam differunt unitatibus. Sed prima earum vel minima, non erit unitas, sed $\frac{1}{1+2} \sqcap \frac{1}{3}$. Ordinatae autem erunt harum abscissarum radices. Si m sit 4. $\sqrt{\frac{m^2}{1+2m}}$ erit $\frac{[4]}{3}$.

$$\frac{2m^2}{1+2m}+1\ \boxed{-2\sqrt{\frac{m^4}{1+4m+4m^2}+\frac{m^2}{1+2m}}}\ -\ 2\sqrt{\frac{m^4+m^2+2m^3}{1+4m+4m^2}} \sqcap z^2 .$$

Iam ex hac quantitate signo affecta potest extrahi radix, fiet ergo $\frac{2m^2}{1+2m}+1-2\frown\frac{m^2+m}{1+2m} \sqcap z^2$, fietque $1-\frac{2m}{1+2m} \sqcap z^2$, sive $\frac{1\boxed{+2m-2m}}{1+2m}$. et fiet $z \sqcap \sqrt{\frac{1}{1+2m}}$. quod conferendum superiori $z \sqcap \sqrt{2y+1-2\sqrt{y^2+y}}$, sive $z \sqcap \sqrt{\boxed{\frac{2m^2}{1+2m}}+1\boxed{-\frac{2m^2}{1+2m}}-2m}$. Rursus error in calculo.

Quem ut corrigamus videndum: $\frac{m^2}{1+2m}+1,\frown\frac{m^2}{1+2m}$, facit $\frac{m^4}{1+2m,\square}+\frac{m^2}{1+2m}$, vel $\frac{m^4,+m^2\frown 1+2m}{1+2m,\square}$ vel $\frac{m^4+m^2+2m^3}{1+2m,\square}$ vel $\frac{m^2+m}{1+2m},\square$.

Ergo $-2\sqrt{\frac{[m^2]}{1+2m}+1\frown\frac{m^2}{1+2m}}$ facit: $-2\sqrt{\frac{m^2+m}{1+2m},\square} \sqcap -2\frown\frac{m^2+m}{1+2m}$.

Ergo $-2\ \sqrt{y^2+y} \sqcap \frac{-2m^2-2m}{1+2m}$ et $\sqrt{y^2+y} \sqcap \frac{m^2+m}{1+2m}$. Quod debet esse $y-m$. seu $\frac{m^2}{1+2m}-m$ seu $\frac{\boxed{m^2}-m-\boxed{2}m^2}{1+2m}$. Sola ergo in signis differentia est. Quodsi ergo ponamus $m-y$, fiet $\frac{-m^2+m+2m^2}{1+2m}$ sive $\frac{m^2+m}{1+2m}$. Quod et faciendum est nam quia

4 vel minima *erg. L* 5 2 *L ändert Hrsg.* 13 m^2+1 *L ändert Hrsg.*

$y \sqcap \dfrac{m^2}{1+2m}$ ideo y est minor quam m. Concors ergo hoc modo fit calculus. Quia ergo $\sqrt{y^2+y} \sqcap \dfrac{m^2+m}{1+2m}$. ideo pro $z \sqcap \sqrt{2y+1-2\sqrt{y^2+y}}$. fiet:

$z \sqcap \sqrt{\boxed{\dfrac{2m^2}{1+2m}}+1\boxed{-\dfrac{2m^2}{1+2n}}-\dfrac{2m}{1+2m}}$, et vel $z \sqcap \sqrt{1-\dfrac{2m}{1+2m}}$, sive $z \sqcap \sqrt{\dfrac{1+2m-2m}{1+2m}}$, sive $z \sqcap \dfrac{1}{\sqrt{1+2m}}$.

Quod si pro $\sqrt{y^2+y}$ ponamus radicem, $\dfrac{m^2}{1+2m}-m$, quod in nostra potestate est, fiet: $z^2 \sqcap \boxed{\dfrac{2m^2}{1+2m}}+1\boxed{-\dfrac{2m^2}{1+2m}}+2m$, et erit $z \sqcap \sqrt{1+2m}$.

Hinc sequitur radices numerorum imparium supra unitatem summari posse, quantocunque sint numero, per compendium regulae generalis.

Sed hinc sequeretur et diversas ipsarum z. significationes habere summas inter se invicem aequales, quod est absurdum. Itaque radix $\sqrt{y^2+y}$ non debet aequari quantitati negativae, alioquin, foret $-\sqrt{y^2+y}$ additio potius quam subtractio, ac proinde idem ac si sumsissemus non $\sqrt{y+1}-\sqrt{y}$ sed $\sqrt{y+1}+\sqrt{y}$. Posterior ergo acceptio non procedit, nec z valere potest $\sqrt{1+2m}$.

Quod si faciamus y^2+y, a $y^2+2yn+n^2$. fiet $y \sqcap \dfrac{n^2}{1-2n}$. ac proinde ponendo y esse quantitatem affirmativam, fiet: duplum ipsius n, minus unitate et fiet $z^2 \sqcap \boxed{\dfrac{2n^2}{1-2n}}+1$ $\boxed{-\dfrac{2n^2}{1-2n}}-2n$ et $z \sqcap \sqrt{1-2n}$.

Atque ita sumendo seriem ipsarum n, arithmeticae progressionis, sed ita, ut maxima non perveniat ad $\dfrac{1}{2}$. summa earum iniri potest.

Ut sciamus an ope alicuius seriei haberi possit summa infinitarum eius differentiarum, ecce notam[:] Si minima seriei ordinata assignari potest; tunc series differentiarum non

8 f. generalis. *(1)* Tantum superest experiamur, quid fiat *(2)* Sed *L*

nisi finita haberi potest, v. g. si minima $\sqrt{y}$ seu minima $\sqrt{\dfrac{m^2}{1+2m}}$ ut habeatur, ponamus $m \sqcap$ quantumlicet parvam, et postea paulo maiorem. Si ipsius m seu quantitatis naturalis, minutio minuet, auctio auget ordinatam, tunc series est crescens, et habetur minima ordinata; sin contra; series est decrescens et haberi ea non potest. Si series habet flexum contrarium, quasi duae concipiendae sunt series, inter se per flexum contrarium divisae, quantum ad nostrum scopum.

Nota etsi dixi 2 debere esse minus quam n, attamen si ab initio sumsissemus, non $\sqrt{y+1} - \sqrt{y}$, sed $\sqrt{y+a} - \sqrt{y}$, explicando v. g. a per 1000. seu $\sqrt{y+1000} - \sqrt{y}$. fiet: $\sqrt{y+a} - \sqrt{y} \sqcap z$. et $2y + a - 2\sqrt{y^2+ay} \sqcap z^2$. et ponendo $y^2 + ay \sqcap y^2 + 2ny + n^2$, fiet $y \sqcap \dfrac{n^2}{a-2n}$. et $-2\sqrt{y^2+ay} \sqcap -2y - 2n \sqcap \dfrac{-2n^2}{a-2n} - 2n$. et fiet $z^2 \sqcap \boxed{\dfrac{2n^2}{a-2n}} + a \boxed{-\dfrac{2n^2}{a-2n}} - 2n$. sive $z \sqcap \sqrt{a-2n}$. Posito ergo $a \sqcap 1000$, sufficit n esse infra 500.

Sed hinc iam video id fieri non posse, nam necesse est n, esse invariabiles; alioquin non potest differentia duarum diversarum ordinatarum esse: $\sqrt{\dfrac{n^2}{a-2n} - a}, -\sqrt{\dfrac{n^2}{a-2n}}$, idem est supra de ipsis m. quare falsa sunt omnia quae de seriebus variabilibus $\dfrac{1}{\sqrt{1+2m}}$, vel $\sqrt{a-2n}$. ineundis diximus: Nec proinde valor ipsius y explicari debet et quaerendus est casus, in quo valor cuiusdam quantitatis in valorem incognitae proximae, eodem modo formatae ductus, det quadratum. Eiusque casus ope simplicium radicum haberi poterit summa:

Aliter ponendo x ordinatam, et ξ proxime minorem ordinatam faciendoque $x - \xi \sqcap z$, et $x^2 - 2x\xi + \xi^2 \sqcap z^2$. Si iam $2x\xi$ est quadratus, vel quod vix putem eventurum destrui

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12 *Am Rande:* ☽

2 f. naturalis, *(1)* | minutio *streicht Hrsg.* | auget *(2)* minutio *L* 6 f. scopum. *(1)* Habemus ergo duo inventa admiranda: Compen *(2)* Nota *L* 15 f. est *(1)* quasi *(2)* casus, in quo *(a)* ordinatae qua *(b)* quadratum ordinatae in quadratum ordinatae proximae ductum det quadrat *(c)* quan *(d)* formula quaedam in *(e)* valor *(aa)* quanti *(bb)* cuiusdam *(aaa)* incognitae *(bbb)* quantitatis *L* 19 Aliter *(1)* si efficiatur ut *(a)* $\sqrt{\mathrm{y}}$ *(b)* $\sqrt{\mathrm{x}}\ \widehat{}\ \mathrm{x}$ pro *(2)* ponendo *(a)* $\sqrt{\mathrm{x}}$ ordinatam et $\sqrt{\mathrm{x}+\mathrm{w}}$. ordinatam | proximam *streicht Hrsg.* |, *(aa)* si $-2\mathrm{x} + \mathrm{xw} - 2\mathrm{xw} + \mathrm{x}^2 +$ *(bb)* | si sit *streicht Hrsg.* | $\mathrm{x} + \mathrm{w} - \mathrm{x}$, *(aaa)* et qu *(bbb)* $\sqcap \mathrm{z}$ ($\sqcap$ w.) | et inde *streicht Hrsg.* | $\mathrm{x}^2 + 2\mathrm{xw} + \mathrm{w}^2 + \mathrm{w}^2 - 2\mathrm{wx} \sqcap \mathrm{z}^2$. *(b)* x ordinatam *L*

potest, radicum simplicium haberi potest summa. Ut $x\xi$ sit quadratus necesse est inveniri posse seriem numerorum non quadratorum, ita tamen ut quilibet ex ipsis ductus in proximum det quadratum.

$$\begin{array}{ccccccccc} a & & b & & c & & d & & e \\ & ab \sqcap l^2 & & bc \sqcap m^2 & & cd \sqcap n^2 & & de \sqcap p^2 & \end{array}$$

Ergo $a \sqcap \frac{l^2}{b}$, et $b \sqcap \frac{m^2}{c}$, et $c \sqcap \frac{n^2}{d}$, et $d \sqcap \frac{p^2}{e}$. Ergo $c \sqcap \frac{n^2 e}{p^2}$, et $b \sqcap \frac{m^2 p^2}{n^2 e}$, et $a \sqcap \frac{l^2 n^2 e}{m^2 p^2}$.

$\frac{3}{1}\quad \frac{4}{3}\quad \frac{27}{4}\quad \frac{[64]}{27}\quad \left[\frac{27 \frown 25}{64}\right]$ etc. Eodem modo incipi poterat a $\frac{2}{1}$ et fieret: $\frac{2}{1}\quad \frac{4}{2}\quad \frac{18}{4}$ etc.

Vide partem VII[am] seu plag. seq.

2 ut *(1)* duo *(2)* duo *(3)* quilibet *L*   8 100 *L ändert Hrsg.*   8 $\frac{27 \frown 36}{100}$ *L ändert Hrsg.*

38_9. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS SEPTIMA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 16–17. 1 Bog. 2°. 4 S. Geringer Textverlust durch Randschäden.
Cc 2, Nr. 775 A tlw.

Schediasmatis de serierum summis pars VII[ma].

Esto series eiusmodi:

$$\frac{f}{y^2}.\qquad \frac{y^2, \frown y^2 \mp 2y\beta + \beta^2}{f}.\qquad \frac{f, \frown y^2 \mp 4y\beta + 4\beta^2}{y^2, \frown y^2 + 2y\beta + \beta^2}.$$

$$\frac{y^2, \frown y^2 \mp 2y\beta + \beta^2, \frown y^2 \mp 6y\beta + 9\beta^2}{f, \frown y^2 \mp 4y\beta + 4\beta^2}. \text{ etc.}$$

Necesse est autem f esse quantitatem non quadratam.

Si $\mp$ significat $+$ series crescit si $\mp$ significet $-$ series decrescit, contrarium evenit si ex serie exposita aliam facias invertendo terminos quoslibet, faciendo numeratorem ex nominatore et contra, ut sit:

$$\frac{y^2}{f}.\qquad \frac{f}{y^2, \frown y^2 \mp 2y\beta + \beta^2}.\qquad \frac{y^2, \frown y^2 + 2y\beta + \beta^2}{f, \frown y^2 \mp 4y\beta + 4\beta^2}.$$

$$\frac{f, \frown y^2 \mp 4y\beta + 4\beta^2}{y^2, \frown y^2 \mp 2y\beta + \beta^2, \frown y^2 \mp 6y\beta + 9\beta^2}.$$

Non recte formavi, ne opus sit mutare formam, seu numeros, rectius sic fiet:

$\frac{b^2 \frown y^2}{f}$. $\frac{f, \frown y^2 \mp 2y\beta + \beta^2}{b^2 \frown y^2}$. Ecce duo termini quibus expositis exponitur tota series, nam intelligi debet, tertium fieri ex secundo, ut secundus fit ex primo. Sed mutanda nonnihil formula est, ut omnia comprehendat et fiet ita:

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19 f. *Zur Streichung:* Error

5 f. pars *(1)* VI[ta] *(2)* VII[ma]. | Inveni tandem methodum, qua haberi possit *(a)* series num *(b)* summa seriei numerorum irrationalium, simplicium, tam finitorum quam infinitorum. *gestr.* | Esto *L*

14 f. $\frac{\mathrm{f}, \frown \mathrm{y}^2 \mp 4\mathrm{y}\beta + 4\beta^2}{\mathrm{y}^2, \frown \mathrm{y}^2 \mp 2\mathrm{y}\beta + \beta^2, \frown \mathrm{y}^2 \mp 6\mathrm{y}\beta + 9\beta^2}$. *(1)* Sit alia *(2)* Sint aliae *(a)* radices *(b)* series duae *(3)* | Sint aliae duae series radicum ab his terminis: *streicht Hrsg.* | *(4)* Imo r *(5)* Non *L* 15 numeros *(1)* sufficit *(2)*, rectius *L* 17 primo. *(1)* Hoc uno tantum adiecto quod formulae ipsi commode inseri non potest. *(2)* Sed *L*

$$\frac{A \sqcap \square}{B \sqcap \text{non } \square} \qquad \frac{C \sqcap B \frown y^2}{D \sqcap A} \qquad \frac{E \sqcap D \frown y^2 \text{ ǂ } 2y\beta + \beta^2}{F \sqcap C}$$

Si iam seriem continuare velis, tertius pro primo sumendus, quartus pro secundo, quintus pro tertio. Idem est, si omnes terminos invertas.

Posito seriem directam esse decrescentem, ob ǂ non potest tamen decrescere in infinitum, nam infra primum y descendi non potest in numeris. Contrarium est in serie inversa.

Nunc ut veniam ad summam irrationalium, utar numeris

$$\sqrt{\frac{1}{2}} \qquad \sqrt{\frac{2}{1}} \qquad \sqrt{\frac{4}{2}} \qquad \sqrt{\frac{18}{4}} \qquad \sqrt{\frac{64}{18}} \qquad \sqrt{\frac{450}{64}} \quad \text{etc.}$$

$$0 \qquad \sqrt{\frac{1}{2}} \qquad -\sqrt{\frac{1}{18}}$$

Series autem differentiarum inter has radices, ita habebuntur: $\sqrt{\frac{4}{2}} - \sqrt{\frac{2}{1}} \sqcap w$, fiet: $w^2 \sqcap \frac{4}{2} + \frac{2}{1} - 2\sqrt{\frac{4 \frown 2}{2 \frown 1}} \sqcap \frac{4}{2} + \frac{2}{1} - 4 \sqcap 0$. Ergo $w \sqcap \sqrt{0}$. Eodem modo: $\sqrt{\frac{18}{4}} - \sqrt{\frac{4}{2}} \sqcap w$.

8–455,2 *Nebenbetrachtungen und Nebenrechnungen:*

8

$$\begin{array}{r} 25 \\ \underline{18} \\ 200 \\ \underline{25} \\ 450 \end{array} \qquad \begin{array}{l} \not{2} \\ \not{1}\not{8}\not{0}0 \;⨍\; 450 \\ \not{4}\not{4} \end{array}$$

9 $\frac{3}{2}\sqrt{2}, -\frac{4}{3}\sqrt{\frac{4}{2}}$

2 sumendus, *(1)* secundus *(2)* quartus *L* 4 directam *erg. L* 4 ob ǂ *erg. L*

et $w^2 \sqcap \frac{18}{4} + \frac{4}{2} - 2 \boxed{\sqrt{\frac{18,4}{4,2}}}, \frown 3$. Ergo $w \sqcap \sqrt{\frac{1}{2}}$. Eodem modo: $w^2 \sqcap \boxed{\frac{64}{18}} \underset{4+\frac{1}{2}}{\boxed{+\frac{18}{4}}} - 2 \frown$ $4 \boxed{+\frac{72}{18}} - \frac{8}{18} \sqcap \frac{1}{18}$. Ergo $w \sqcap \sqrt{\frac{1}{18}}$. Ita $\boxed{\frac{450}{64}} \boxed{+\frac{64}{18}} \boxed{-2 \frown 5} + \overset{\boxed{5}}{\boxed{\frac{90}{18}}} - \frac{16}{18} \boxed{+5} + \frac{130}{18} \sqcap \frac{114}{18}$.

---

1 $\frac{18}{4} + \frac{4}{2} - 2 \frown 3 \sqcap \frac{1}{2}$ $\frac{64}{18} + \frac{18}{4} \sqcap \frac{64 \frown 4 + 18 \frown 18}{18 \frown 4}$ $\frac{64}{18} + \frac{18}{4} - 2 \frown 4 \sqcap 8$

18	
18	
144	
18	
324	
256	72
580	8
576	576
4	

2 $\frac{450}{64}$ $\frac{64}{18} - \not{2} \frown 10 \sqcap$

450	64	64	8100	4	
18	64	18	4096	$\not{5}8$	
360	256	512	12196	$\not{1}\not{1}\not{5}20$	⨍ 17
45	384	64	11520	$\not{6}\not{7}66$	
8100	4096	11520	676	67	

18	64	450	130
5	5	320	16
90	320	130	114

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2 $-\frac{16}{18} \ldots + \frac{130}{18}$: Richtig wäre $-\frac{26}{18} + \frac{130}{64} = \frac{169}{288}$.

Generaliter $\sqrt{\frac{f}{y^2}} - \sqrt{\frac{y^2, \frown y^2 + 2\beta y + \beta^2}{f}} \sqcap w$. Ergo $w^2 \sqcap \frac{f}{y^2} + \frac{y^2, \frown y^2 + 2\beta y + \beta^2}{f} - 2\sqrt{y^2 + 2y\beta + \beta^2}$ seu $-2 \frown y + \beta$. Ergo $w^2 \sqcap \frac{f}{y^2} + \frac{y^2, \frown y^2 + 2\beta y + \beta^2}{f} - 2y - 2\beta$. sive $\frac{f^2 + \quad y^4, \frown y^2 + 2\beta y + \beta^2, \quad -2fy^3 - 2y^2\beta f}{y^2 f}$. Sed hinc iam video istam seriem reapse non esse seriem irrationalium, sed seriem rationalium, per quandam constantem $\sqrt{f}$ divisam, nam ex $\sqrt{\frac{f^2 + \quad y^4, \frown y^2 + 2y\beta + \beta^2 \quad - 2fy^3 - 2fy^2\beta}{y^2 f}}$ per $\sqrt{f}$ multiplicata, radix extrahi potest, fiet enim eius radix $\frac{f, [-]y^2 \frown y + \beta}{y}$.

Nondum ergo methodum video, per quam summae iniri possint irrationalium simplicium seu naturalium.

$$\frac{1}{1,2} \quad \frac{1}{2,3} \quad \frac{1}{3,4} \quad \frac{1}{4,5} \quad \frac{1}{5,6} \quad \text{etc. seu } \frac{1}{y \frown y+1}.$$

Momentum eorum ex y. est series omnium $\frac{1}{y+1}$. Momentum eorum ex $y+1$. est series omnium $\frac{1}{y}$. Momentum autem ex $y+1$. a momento ex y. differt serie in y. seu seriei cylindro. At series omnium $\frac{1}{y}$ excedit seriem omnium $\frac{1}{y+1}$ maxima $\frac{1}{y}$. Ergo maxima $\frac{1}{y} \sqcap$ seriei omnium $\frac{1}{y, \frown y+1}$.

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8 *Am Rande, gestrichen:* Hactenus nihil egi in summis irrationalium simplicibus investigandis. Sed iam esto series haec: $\sqrt{2}. + 2\sqrt{2}.$
$\sqrt{2}$

4 quandam *(1)* rationalem *(2)* constantem *L* 6 + *L ändert Hrsg.* 10 f. series omnium *erg. L zweimal* 11 differt *(1)* figura *(2)* serie *L* 12 cylindro. *(1)* Differentia ergo inter *(2)* At idem moment *(3)* At *L* 12 maxima *(1)* serierum *(2)* $\frac{1}{y}$. Ergo *L*

Eadem methodo omnium habetur dimensio quae sunt eiusmodi v. g. $\frac{1}{y, \frown y+1, \frown y+2}$. Nam momentum eorum ex y. est $\frac{1}{y+1, \frown y+2}$, momentum eorum ex $y+2$. est $\frac{1}{y, \frown y+1}$, ergo quae duo momenta differunt duplo seriei cylindro; differunt vero et cognito seu termino seriei $\frac{1}{y, \frown y+1}$, qui non continetur in terminis seriei $\frac{1}{y+1, \frown y+2}$, id est primo.

Sed et praeterea, momentum ex $y+1$, est $\frac{1}{y, \frown y+2}$. Quod momentum a priori ex $y+2$. cedit figurae cylindro simplici. Quem invenimus.

Datur vero et momentum ex $y+2$. absolute, seu $\frac{1}{y, \frown y+1}$. Ergo datur et series omnium $\frac{1}{y, \frown y+2}$ seu $\frac{1}{1,3}\ \frac{1}{2,4}\ \frac{1}{3,5}\ \frac{1}{4,6}\ \frac{1}{5,7}$ etc. Pro 1. pone β habetur series: (1) $\frac{1}{y, \frown y+\beta}$. Et series (2) $\frac{1}{y, \frown y+\beta, \frown y+2\beta}$ et (3) $\frac{1}{y, \frown y+2\beta}$.

Datur vero et series: (4) $\frac{1}{y, \frown y+\beta, \frown y+2\beta, \frown y+3\beta}$. Dabitur ergo et series:

(5) $\frac{1}{y, *, \frown y+2\beta, \frown y+3\beta}$, ut $\frac{1}{1,3,4}\quad \frac{1}{2,4,5}\quad \frac{1}{3,5,6}\quad \frac{1}{4,6,7}$.

$\frac{1}{12}\quad \frac{1}{40}\quad \frac{1}{90}\quad \frac{1}{168}$

Ex hac autem inventa rursus eodem ratiocinandi modo, datur (6) $\frac{1}{y, \frown y+3\beta}$ seu $\frac{1}{4}\ \frac{1}{10}\ \frac{1}{28}$ per datas 3. et 1.

2 $\frac{1}{y, \frown y+1, \frown y+2}$ *(1)* $-\frac{1}{y^2}$ *(2)*. Nam cylinder eorum ex *(a)* y est $\frac{1}{y+1, \frown y+2}$. *(b)* y + 2. est $\frac{1}{y, \frown y+1}$. Cylinder eorum ex y + 1. est $\frac{1}{y, \frown y+2}$ *(3)* Cylinder eorum ex y + 2. est $\frac{1}{y, \frown y+1}$ *(4)* Nam *(a)* cylinder *(b)* momentum *L* 2 $\frac{1}{y+1, \frown y+2}$, *(1)* cylinder *(2)* momentum *L* 4 et *(1)* recta *(2)* cognito *L* 6 praeterea, *(1)* idem mo *(2)* momentum *L* 7 y + 2, *(1)* differet *(2)* cedit *(a)* etiam *(b)* figurae *L*

Ex data 4. eodem modo quo 5. dabitur et series $\dfrac{1}{y, \frown y+\beta, *, \frown y+3\beta}$, seu

$$\frac{1}{1,2,4} \qquad \frac{1}{2,3,5} \qquad \frac{1}{3,4,6}.$$
$$\frac{1}{8} \qquad \frac{1}{30} \qquad \frac{1}{72}$$

Generaliter ergo dici potest, omnes series quarum numeratores sunt constantes, nominatorum vero factores arithmetice excedunt perpetua progressione factores nominatoris fractionis praecedentis, esse summabiles.

Quid ergo in hac serie
$$\frac{1}{3} \qquad \frac{1}{35} \qquad \frac{1}{99}$$
$$\frac{1}{1,3} \qquad \frac{1}{5,7} \qquad \frac{1}{9,11}$$
$$\frac{1}{y \frown y+2\beta} \qquad \frac{1}{y+4\beta, y+6\beta}$$
cuius seriei ut investigemus summam, conemur eam reducere ad eas quas habemus.

Ut uno termino exprimamus, ponendo β. semper esse terminum constantem, per quem crescunt ipsae y. fiet: $\dfrac{1}{y, \frown y+\frac{1}{2}\beta}$, ponendo β. esse 4. sive $\dfrac{2}{y, \frown 2y+\beta}$, cuius dimidium $\dfrac{1}{y, \frown 2y+\beta}$.

Adhibeamus ergo alias series, ut: $\dfrac{1}{y, \frown y+\frac{1}{2}\beta, \frown y+\beta}$. Eius momentum ex $y+\frac{1}{2}\beta$. datur sed caetera eius momenta non dantur.

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7 *Am unteren Rand, mit Verbindungsstrich zu* hac serie: Mylord Brouncker dedit summam huius seriei: $\dfrac{1}{2,3,4} \quad \dfrac{1}{4,5,6} \quad \dfrac{1}{6,7,8}$ in *Transact.* num. 34. pag. 646.

10 summam, *(1)* et 2β. *(2)* conemur *(a)* terminos *(b)* eam *L* 10 f. habemus. *(1)* Pro 2β. ponamus *(2)* Ut *L* 11 constantem *erg.* *L* 12 esse 4. *(1)* quare duplicata tota serie fiet: *(2)* sive *L* 15 datur *(1)*. Datur et eius momentum ex *(2)* sed *L*

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16 Brouncker: *The Squaring of the Hyperbola, by an Infinite Series of Rational Numbers, Philosophical Transactions* III Nr. 34 vom 13./23. April 1668, S. 645–649.

Si figura sit: $\dfrac{1}{y, \frown 2y+\beta}$ eius momentum ex y. est $\dfrac{1}{2y+\beta}$, v. g. $\dfrac{1}{3}\ \dfrac{1}{5}\ \dfrac{1}{7}$. si β. sit 1. Eius momentum ex $2y+\beta$, est $\dfrac{1}{1}\ \dfrac{1}{2}\ \dfrac{1}{3}$. Ipsa autem series $\dfrac{1}{y, \frown 2y+\beta}$ est $\dfrac{1}{3}\ \dfrac{1}{10}\ \dfrac{1}{21}$. nempe triangulares per intervalla.

A serie $\dfrac{1}{2}\ \dfrac{1}{3}\ \dfrac{1}{4}\ \dfrac{1}{5}\ \dfrac{1}{6}\ \dfrac{1}{7}$ auferendo $\dfrac{1}{3}\ \dfrac{1}{5}\ \dfrac{1}{7}$, residuum erit figurae momentum ex y. etc. sed nihil hinc novi nisi hoc unum, quod series $\dfrac{1}{3} * \dfrac{1}{10} * \dfrac{1}{21}$ pendeat ex quad. circuli.

Iam $\dfrac{1}{1}\ \dfrac{1}{6}\ \dfrac{1}{15}\ \dfrac{1}{28}$ fit $\dfrac{1}{1}-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}+\dfrac{1}{5}-\dfrac{1}{6}$ duplicata, quae pendet ex quadratura hyperbolae. Summa autem utriusque habetur, itaque hinc videtur sequi quadraturam circuli pendere ex quad. hyperbolae et contra. Sed hoc postea examinabimus, nunc cogitationem subortam persequar: $\dfrac{1}{y, \frown y^2+2y\beta+\beta^2, -y^3}$ potest intelligi facta ex $\dfrac{1}{y, 2y+\beta, \beta}$ ac proinde haec pendet etiam ex circuli dimensione.

Si sint iam series eiusmodi:
$\dfrac{1}{y^2, \frown y^2+2\beta y+\beta^2}$, vel $\dfrac{1}{y^2, \frown y^2+2\beta y+\beta^2, \frown y^2+4\beta y+4\beta^2}$, videamus. Momentum prioris ex y dat: $\dfrac{1}{y, y+\beta, y+\beta}$ cuius valorem primo investigemus. Eius momentum ex y

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12 *Neben* videamus: Momentum prioris ex $y+\beta$ dat: $\dfrac{1}{y^2, \frown y+\beta}$ cuius dimensio pendet ex $\dfrac{1}{y^2}$.

5 f. circuli *(1)* $\dfrac{1}{3}-\dfrac{1}{5}+\dfrac{1}{7}$ *(2)*. Iam *L* 6 duplicata *erg.* *L* 9 $\dfrac{1}{\mathrm{y}, \frown \mathrm{y}^2+2\mathrm{y}\beta+\beta^2, -\mathrm{y}^3}$ *(1)* . *(a)* Haec ducta *(b)* Horum momen *(2)* potest *L* 10 f. dimensione. *(1)* Ergo differentia inter *(2)* Si *L*

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5 pendeat: Leibniz verwechselt die Reihe $\dfrac{1}{3}+\dfrac{1}{5}+\dfrac{1}{7}$ etc. mit der Kreisreihe in der Form $\dfrac{1}{3}-\dfrac{1}{5}+\dfrac{1}{7}$ etc. Die Überlegung wird dadurch bis Z. 10 beeinträchtigt. 8 postea: s. u. S. 462 Z. 3.

est $\frac{1}{y+\beta,\square}$. Eius momentum ex $y+\beta$, est $\frac{1}{y^2+\beta y}$ quae habetur. Patet ergo has duas figuras $\frac{1}{y,,\ y+\beta,\square}$ et $\frac{1}{y+\beta,\square}$ pendere ex se invicem.

Si sit series: $\frac{1}{y,y+\beta,y-\beta}$, momentum eius ex y. est $\frac{1}{y^2-\beta^2}$, quod habetur. Momentum eius ex $y-\beta$. etiam habetur. Quare habetur ipsa series: $\frac{1}{y,y+\beta,y-\beta}$. Hinc patet ipsas $y-\beta$. ipsis $y+\beta$. tuto immisceri posse, hac serie: $y-\beta,y,y+\beta$. Ita enim servatur arithmeticus ordo.

Iam in seriem: $\frac{1}{y^2-\beta^2}$ ducatur: $\frac{1}{y+\beta}$, fiet $\frac{1}{y-\beta,,\frown\ y+\beta,\square}$, quae pendet ex ista $\frac{1}{y^2}$. Nam momentum eius ex $y+\beta$. habetur, momentum vero eius ex $y-\beta$. est $\frac{1}{y+\beta,\square}$, vel $\frac{1}{y^2}$. $y^2-\beta^2 \frown y^2+\beta^2$ facit $y^4-\beta^4$. $y^2-\beta \frown y^2+\beta$ dabit $y^4-\beta^2$.

Videamus an non $\frac{1}{y^2,\frown\ y+\beta,\square}$, possit haberi absolute, quadrata scilicet triangularium. Eius momentum ex y est $\frac{1}{y,\frown\ y+\beta,\square}$, eius momentum ex $y+\beta$, est $\frac{1}{y^2,y+\beta}$. Horum differentia est cylinder seriei. Porro huius $\frac{1}{y+\beta,\square,y}$, momentum ex $y+\beta$, est series triangularis, momentum eiusdem ex [y], est $\frac{1}{y+\beta,\square}$. Ergo ser. $\triangle^{\text{lar.}} - \frac{1}{y+\beta,\square} \sqcap \frac{1}{y,,y+\beta,\square}$. Alterius vero: $\frac{1}{y^2,y+\beta}$, momentum ex y est series triang. ex $y+\beta$. est $\frac{1}{y^2}$. Ergo $\frac{1}{y^2} -$ ser. $\triangle^{\text{lar.}} \sqcap \frac{1}{y^2,y+\beta}$.

1 ergo *(1)* momentum *(2)* has *L* 9 vel $\frac{1}{\mathrm{y}^2}$. *(1)* Momentum eius ex *(2)* $\mathrm{y}^2-\beta^2 \frown \mathrm{y}^2+\beta^2$ *L*
13 $\frac{1}{\mathrm{y}}$ *L ändert Hrsg.* 13 Ergo *(1)* $\frac{1}{\mathrm{y},\mathrm{y}+\beta}$ *(2)* ser. $\triangle^{\text{lar.}}$ *L*

10 f. quadrata ... triangularium: s. o. Erl. zu N. 38₇ S. 438 Z. 11 f.

Iam, ex dictis: $\frac{1}{y^2, y+\beta} - \frac{1}{y,,y+\beta,\square} \sqcap \frac{1}{y \frown y+\beta,\square}$. Ergo substitutis valoribus: $\frac{1}{y^2} -$ ser. $\triangle^{\text{lar.}}$, $-$ ser. $\triangle^{\text{lar.}} + \frac{1}{y+\beta,\square} \sqcap \frac{1}{y \frown y+\beta,\square}$. Iam supra plag. huius schediasm. 5. $\frac{1}{y^2} \sqcap C + \frac{1}{b}$. et $C \sqcap \frac{1}{2b^2} + \frac{B}{2}$. et B est $\frac{1}{y, \frown y+\beta,,\square}$. Ergo $\frac{\cancel{1}2}{y^2} \sqcap \frac{1}{\cancel{2}b^2} + \frac{B}{\cancel{2}} + \frac{\cancel{1}2}{b}$. Iam $\frac{1}{y^2} + \frac{1}{y+\beta,\square} \sqcap \frac{2}{y^2} - \frac{1}{b^2}$. ergo $B \sqcap [-]2$ ser. $\triangle^{\text{lar.}} + \frac{[2]}{y^2} - \frac{1}{b^2}$. quo in aequatione praecedente novissima, pro B substituto fiet: 2 ser. $\triangle^{\text{lar.}} \sqcap \frac{2}{b}$. quod dudum constat. Indicium ⟨calculi recti.⟩

$\frac{1}{y^2 \frown y^2-\beta^2}$ puto haberi posse. Nam: eius momentum ex $y+\beta$, est $\frac{1}{y^2, y-\beta}$. Eius momentum ex $y-\beta$, est $\frac{1}{y^2, y+\beta}$. Ergo $\frac{1}{y^2, y-\beta} - \frac{1}{y^2, y+\beta} \sqcap \frac{2}{y^2, y^2-\beta^2}$ (1). Iam: $\frac{1}{y^2, y-\beta}$ ex $y-\beta$, dat $\frac{1}{y^2}$, ex y dat $\frac{1}{y, y-\beta}$. Ergo $\frac{1}{y, y-\beta} - \frac{1}{y^2} \sqcap \frac{1}{y^2, y-\beta}$ (2).

Eodem modo: $\frac{1}{y^2, y+\beta}$ per $y+\beta$ dat $\frac{1}{y^2}$, per y dat $\frac{1}{y, y+\beta}$. Ergo (3) $\frac{1}{y^2} - \frac{1}{y, y+\beta} \sqcap \frac{1}{y^2, y+\beta}$. Ergo in aeq. 1. substitutis valoribus ex aeq. 2. et 3. fiet:

$$\frac{1}{y, y-\beta} - \frac{1}{y^2} - \frac{1}{y^2} + \frac{1}{y, y+\beta} \quad \sqcap \frac{2}{y^2, y^2-\beta^2}, \text{ sive:}$$

$$\frac{2}{y, y+\beta} + \frac{1}{b+\beta, b+2\beta} - \frac{2}{y^2} \quad \sqcap \frac{2}{y^2, y^2-\beta^2}. \text{ Hinc illud saltem ducimus:}$$

$$\frac{2}{y^2, y^2-\beta^2} + \frac{2}{y^2} \boxed{- \frac{2}{y, y+\beta}} - \frac{2}{b} \sqcap \frac{1}{b+\beta, b+2\beta}, \text{ sive}$$

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7 *Über* haberi posse: Imo non.

4 − *erg. Hrsg.* 4 1 *L ändert Hrsg.*

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2 supra: s. o. N. 38₇ S. 441 Z. 1–3 bzw. die Gleichungen (2) und (5) S. 443 Z. 8. 8–11 Leibniz setzt im Zähler der rechten Seite der Gleichungen (1)–(3) stillschweigend $\beta = 1$.

$\frac{2}{y^2, y^2-\beta^2} + \frac{2}{y^2} \sqcap$ cognito $\frac{1}{b+\beta, b+2\beta} + \frac{2}{b}$. Habemus ergo:

$\frac{2y^2,, + 2y^2, y^2-\beta^2}{y^2, y^2-\beta^2, y^2}$ seu $\frac{2,, + 2, y^2-\beta^2}{y^2, y^2-\beta^2} \sqcap$ cognitae $\frac{1}{b+\beta, b+2\beta} + \frac{2}{b}$.

Superest, ut quae de circulo et hyperbola dicta sunt, perficiamus.

Series: $\frac{1}{3}. \frac{1}{35}. \frac{1}{99}$. etc. pendet ex quadratura circuli, et aequatur rationi quadrantis circumferentiae ad diametrum. Ea sic enuntiabitur analytice: $\frac{1}{y, y+\frac{1}{2}\beta}$, ponendo, y. esse arithmetice crescentes; et β. esse incrementum. Primam autem y esse $\sqcap$ 1. et ipsam β. incrementum constans esse 4. Habita autem summa unius $\frac{1}{y, y+\frac{1}{2}\beta}$, secundum hanc quam dixi explicationem, habebitur et summa cuiuslibet, quomodocunque prima seu minima y, et intervallum constans ipsarum y, explicentur.

Quod ita ostendo: series ex quadratura circuli pendens ita erit in numeris:

$$\frac{1}{1, 1+\frac{4}{2}} \qquad \frac{1}{1+4, 1+4+\frac{4}{2}} \qquad \frac{1}{1+8, 1+8+\frac{4}{2}} \quad \text{etc.}$$

$$\frac{1}{3} \qquad \frac{1}{35} \qquad \frac{1}{99}$$

Haec series quadruplicata redibit ad hanc:

$$\frac{1}{\frac{1}{4}, \frac{1}{4}+\frac{1}{2}} \qquad \frac{1}{\frac{1}{4}+1, \frac{1}{4}+1+\frac{1}{2}} \qquad \frac{1}{\frac{1}{4}+2, \frac{1}{4}+2+\frac{1}{2}}.$$

Quae a serie $\frac{1}{y, y+\frac{1}{2}\beta}$ in qua prima $y \sqcap 1$. et $\beta \sqcap 1$. seu

4 aequatur *(1)* circumferentiae, ponendo radium esse unitatem *(2)* rationi *L* 10 ostendo: *(1)* primam y, *(a)* voce *(b)* quaecunque sit vocemus *(2)* Unitatem appellemus γ. Reducamus ad unitatem, quam iam seriem *(3)* series *L* 15 prima *erg. L*

13 quadruplicata: Leibniz teilt beide Faktoren der Nenner der Folgenglieder durch vier, multipliziert also in Wirklichkeit mit 16. Die Überlegung wird dadurch nicht beeinträchtigt.

$\dfrac{1}{1,1+\frac{1}{2}}\qquad \dfrac{1}{2,2+\frac{1}{2}}\qquad \dfrac{1}{3,3+\frac{1}{2}}$ in eo tantum differt, quod prima y. sumta est non 1. sed $\frac{1}{4}$. Auferantur a se invicem, sane: $\dfrac{1}{y,y+\frac{1}{2}\beta}-\dfrac{1}{y+\frac{3}{4},y+\frac{3}{4}+\frac{1}{2}\beta}$ fiet:

$$\frac{y+\frac{3}{4},y+\frac{3}{4}+\frac{1}{2}\beta,-\;y,y+\frac{1}{2}\beta}{y,y+\frac{1}{2}\beta\left[,y+\frac{3}{4}\right],y+\frac{3}{4}+\frac{1}{2}\beta}\;\sqcap\;\frac{\begin{array}{l}\boxed{y^2}+\frac{3}{4}y\boxed{+\frac{1}{2}\beta y}+\frac{9}{16},\boxed{-y^2-\frac{1}{2}\beta y}\\ \qquad+\frac{3}{4}..\qquad\quad+\frac{3}{8}\beta\end{array}}{y,y+\frac{1}{2}\beta\left[,y+\frac{3}{4}\right],y+\frac{3}{4}+\frac{1}{2}\beta}\ \text{sive}$$

$$\frac{\frac{3}{4}y,,+\frac{3}{4}\frown y+\frac{3}{4}+\frac{1}{2}\beta}{y,y+\frac{1}{2}\beta\left[,y+\frac{3}{4}\right],y+\frac{3}{4}+\frac{1}{2}\beta}\;\sqcap\;\frac{\frac{3}{4}}{y+\frac{1}{2}\beta\left[,y+\frac{3}{4}\right],y+\frac{3}{4}+\frac{1}{2}\beta}+\frac{\frac{3}{4}}{y,y+\frac{1}{2}\beta\left[,y+\frac{3}{4}\right]}$$

$$\sqcap\;\frac{1}{y,y+\frac{1}{2}\beta}-\frac{1}{y+\frac{3}{4},y+\frac{3}{4}+\frac{1}{2}\beta}.$$

Habenda est ergo series ista: $\dfrac{1}{y+\frac{1}{2}\beta,y+\frac{3}{4}+\frac{1}{2}\beta}$. Fiet $\dfrac{1}{1+\frac{1}{2},1+\frac{3}{4}+\frac{1}{2}}$ seu $\dfrac{1}{\frac{9}{4}+\frac{9}{8}}$ seu $\dfrac{8}{27}$; et $\dfrac{1}{\underbrace{2+\frac{1}{2}}_{\frac{5}{2}},2+\frac{3}{4}+\frac{1}{2}}$ sive $\dfrac{1}{\frac{25}{4}+\frac{15}{8}}\quad\dfrac{8}{50+15}$ etc. sed haec progressio parum tractabilis.

2 sane: *(1)* $\dfrac{1}{\frac{1}{4},\frac{1}{4}+\frac{1}{2}}$ *(2)* ponendo z pro y et γ pro β | quando non significant 1, fiet: *streicht Hrsg.* | $\dfrac{1}{\mathrm{y,y}+\frac{1}{2}\beta}-\dfrac{1}{\mathrm{z,z}+\frac{1}{2}\gamma}$ *(3)* $\dfrac{1}{\mathrm{y,y}+\frac{1}{2}\beta}$ *L*   5 f. $,\mathrm{y}+\frac{3}{4}$ *erg. Hrsg. fünfmal*

$$\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11}$$ etc. ex circulo.

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8}$$ [etc.] ex hyperbola. Huius duplum

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}$$ etc. Ex qua auferatur praecedens ad circulum, fiet:

$$* - \frac{1}{2} + \frac{2}{3} - \frac{1}{4} \quad * - \frac{1}{6} + \frac{2}{7} - \frac{1}{8} \quad * \quad - \frac{1}{10} + \frac{2}{11}$$ etc. Si contra addatur, fiet:

$$\frac{2}{1} - \frac{1}{2} \quad * - \frac{1}{4} + \frac{2}{5} - \frac{1}{6} \quad * - \frac{1}{8}$$ etc.

Si ab $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$ etc.

auferas $\frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8}$ etc.

fiet : $\frac{1}{2} - \frac{1}{12} + \frac{1}{30} - \frac{1}{56}$ etc.

Porro $\frac{1}{2} \quad \frac{1}{6} \quad \frac{1}{12} \quad \frac{1}{20} \quad \frac{1}{30} \quad \frac{1}{42} \quad \frac{1}{56} \quad \frac{1}{72}$ etc. habentur.

$\frac{1}{2} \quad \frac{1}{12} \quad \frac{1}{30} \quad \frac{1}{56}$ habetur ex quad. hyp.

$1 - \frac{1}{2} \qquad \frac{1}{5} - \frac{1}{6} \qquad \frac{1}{9} - \frac{1}{10}.$

Iam $1.\ \frac{1}{5}.\ \frac{1}{9}$ etc. $\sqcap \frac{b^1}{1} + \frac{b^5}{5} + \frac{b^9}{9} \sqcap \frac{1}{1+y^4}$. Vid. seq. plagulam VIII.

2 etc. *erg. Hrsg.*

12 $\sqcap \frac{1}{1+y^4}$: Leibniz will sagen, daß die Summe der $\frac{1}{1+y^4}$ mit der linken Seite gleich ist; richtig wäre $\frac{1}{1-y^4}$, s. u. N. 38₁₀ S. 465 Z. 11–13.

38_{10}. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS OCTAVA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 18–19. 1 Bog. 2°. 4 S.
Cc 2, Nr. 775 A tlw.

Schediasmatis de serierum summis pars VIII.

$\frac{1}{2}\quad \frac{1}{6}\quad \frac{1}{12}\quad \frac{1}{20}\quad \frac{1}{30}\quad \frac{1}{42}\quad \frac{1}{56}\quad \frac{1}{72}\quad \frac{1}{90}\quad \frac{1}{110},$ etc. habentur absolute $\sqcap$ 1.

$\frac{1}{2}\quad \frac{1}{12}\quad \frac{1}{30}\quad \frac{1}{56}\quad \frac{1}{90}$ [etc.] ex quad. hyperbolae.

$\frac{1}{2} - \frac{1}{12} + \frac{1}{30} - \frac{1}{56} + \frac{1}{90}$ etc. exhibent differentiam circuli et hyperbolae seu residuum quadrantis, detracto spatio hyperbolico dimidiato.

Quare si inveniretur separatim $\frac{1}{2}\ \frac{1}{30}\ \frac{1}{90}$ [etc.], tunc data quad. hyperbolae, daretur circuli quadratura. Iam $\frac{1}{2}\ \frac{1}{30}\ \frac{1}{90}$ etc. resolvemus in $\frac{1}{1} - \frac{1}{2} + \frac{1}{5} - \frac{1}{6} + \frac{1}{9} - \frac{1}{10}$ etc. Cuius seriei quadratricis quaeratur figura. Quaeramus primum figuram seriei quadratricis; $\frac{1}{1}\ \frac{1}{5}\ \frac{1}{9}$ [etc.] sive: $\frac{b}{1} + \frac{b^5}{5} + \frac{b^9}{9}$ etc. quae fit ex summis omnium: $1 + y^4 + y^8 + y^{12}$ etc. quarum quaelibet aequalis huic ordinatae: $\frac{1}{1 - y^4}$.

Eodem modo series quadratrix $\frac{1}{2}\ \frac{1}{6}\ \frac{1}{10}$ etc. originem habet $\frac{b^2}{2} + \frac{b^6}{6} + \frac{b^{10}}{10}$ etc. quae est summa ordinatarum, $y + y^5 + y^9$ etc. $\sqcap \frac{y}{1 - y^4}$. et figura cuius series quadratrix est $\frac{1}{2}\ \frac{1}{30}\ \frac{1}{90}$ etc., erit $\frac{1 - y}{1 - y^4} \sqcap x$. Sed $1 - y^4$, dividi potest per $1 - y^2$, et $1 + y^2$; et

6–12 etc. *erg. Hrsg. dreimal* 8 f. hyperbolae | seu residuum *(1)* circuli *(2)* circumferentiae in unitatem, *(3)* quadrantis ... dimidiato *erg.* |. Quare *L* 13–16 ordinatae: *(1)* $\frac{1}{1 + \mathrm{y}^4}$ ... $\mathrm{y} + \mathrm{y}^5 + \mathrm{y}^9$ etc. $\sqcap \frac{\mathrm{y}}{1 + \mathrm{y}^4}$ ... erit $\frac{1 - \mathrm{y}}{1 + \mathrm{y}^4}$ *(2)* $\frac{1}{1 - \mathrm{y}^4}$ *L* 16 per *(1)* $1 - \mathrm{y}$, et provenit, $1 - \mathrm{y}^3$, *(2)* $1 - \mathrm{y}^2$, et *(a)* $1 - \mathrm{y}^2$ *(b)* $1 + \mathrm{y}^2$ *L*

$1{-}y^2$, dividi potest per $1{-}y$, et fit $1{+}y$. Ergo ducendo $1{+}y^2$, in $1{+}y$, fit: $\dfrac{1}{y^3+y^2+y+1} \sqcap \dfrac{1}{1+y^2 \frown 1+y}$. Sed missa figura $\dfrac{1}{1+y^2, 1+y}$, videamus figuram $\dfrac{1}{1-y,, 1+y, \square}$. Eius figurae momentum, ex $1-y$, est $\dfrac{1}{1+y, \square}$ cuius habetur dimensio cum sit hyperbola cubica. Eius momentum ex $1{+}y$, est $\dfrac{1}{1-y^2}$. horum duorum momentorum summa aequatur duplo figurae cylindro. Habetur ergo huius figurae dimensio, ex data dimensione huius figurae $\dfrac{1}{1-y^2}$.

Opus est ergo ut huius figurae dimensionem quaeramus, quod eadem methodo egregie fiet: nam momentum eius ex $1+y$ dabit $\dfrac{1}{1-y}$, quod est cylinder spatii hyperbolici. Eiusdem momentum ex $1-y$ dabit $\dfrac{1}{1+y}$, qui est alius cylinder spatii hyperbolici, horum duorum momentorum summa, dabit duplum cylindrum ipsius figurae $\dfrac{1}{1-y^2}$. Ergo ut omnia in unum colligamus, cylindri hyperbolici, unus $\dfrac{a^3}{a-y}$ alter $\dfrac{a^3}{a+y}$, aucti cylindro hyperbolae cubicae, $\dfrac{a^3}{a^2+2ay+y^2}$ dant cylindrum figurae $\dfrac{1}{1-y,, \frown 1+y, \square}$, seu

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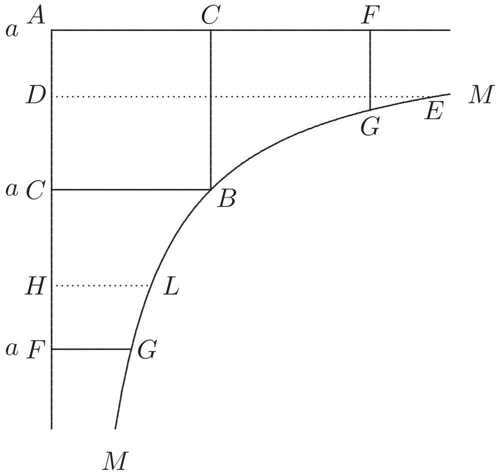
1 *Zur Lesart, gestrichen:* Error.
Error usque ad finem paginae.

1 ducendo *(1)* $1-\mathrm{y}^2$, in $1+\mathrm{y}$, fit: *(a)* $\dfrac{1-\mathrm{y}^2+\mathrm{y}-\mathrm{y}^3}{1-}$ *(b)* $\dfrac{1}{1-\mathrm{y}^2+\mathrm{y}-\mathrm{y}^3} \sqcap \dfrac{1}{1-\mathrm{y}^2, 1+\mathrm{y}} \sqcap \dfrac{1}{1-\mathrm{y},, 1+\mathrm{y}, \square}$. Cuius figurae momentum, *(aa)* est $\dfrac{1}{1-\mathrm{y}}$ *(bb)* ex $1-\mathrm{y}$, *(2)* $1+\mathrm{y}^2$ *L* 3 f. hyperbola *(1)* qua *(2)* cubica *L* 5 duplo *erg. L* 10 duplum *erg. L* 12–467,1 dant *(1)* summam *(a)* figurae, *(b)* cylindri *(2)* cylindrum figurae $\dfrac{\mathrm{a}^5}{\mathrm{a}^3+\mathrm{y}\mathrm{a}^2+\mathrm{y}^2\mathrm{a}+\mathrm{y}^3}$ cuius area aequatur summae $\dfrac{1}{2}$ $\dfrac{1}{30}$ $\dfrac{1}{90}$ etc. Ergo $\dfrac{1}{1-\mathrm{y}} + \dfrac{1}{1+\mathrm{y}} + \dfrac{1}{1+\mathrm{y}, \square} \sqcap \dfrac{1}{2}$ *(3)* cylindrum figurae *(a)* $\dfrac{1}{1+\mathrm{y},, \frown 1+\mathrm{y}, \square}$, seu $\dfrac{1}{1+2\mathrm{y}+\mathrm{y}^2, +\mathrm{y}+2\mathrm{y}^2+\mathrm{y}^3}$ seu $\dfrac{1}{\mathrm{y}^3+3\mathrm{y}^2}$ *(b)* $\dfrac{1}{1-\mathrm{y},, \frown 1+\mathrm{y}, \square}$ *L*

$\dfrac{1}{1+2y+y^2,-y-2y^2-y^3}$ seu $\dfrac{1}{-y^3-y^2+y+1}$. (NB. Iam $\dfrac{1}{y^3+y^2+y+1}$ pendet ex circulo et hyp. simul.) Addendo fiet:

$$\frac{\boxed{y^3+y^2}+y+1\boxed{-y^3-y^2}+y+1}{1+y^2\frown 1+y\frown 1+y,\square,1-y} \sqcap \frac{2}{1+y^2,1+y,1-y} \sqcap \frac{2}{1+y^2\frown 1-y^2} \sqcap \frac{2}{1-y^4}.$$

Cum hoc sit maximi momenti, erroris vitandi causa de integro assumere operae pretium erit.



[*Fig. 1*]

Esto GBE. hyperbola, cuius centrum A. vertex B. Radius ut ita dicam AC. vel $CB \sqcap a$. CD. vel CH. vel $CF \sqcap y$. $AD \sqcap a-y$. AF vel $AH \sqcap a+y$. $DE \sqcap \dfrac{a^2}{a-y}$. $HL \sqcap \dfrac{a^2}{a+y}$. Ergo posita $a \sqcap 1$. erit $DE \sqcap 1+y+y^2+y^3+y^4$ etc. et summa

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3 *Nebenrechnung:* $\dfrac{1}{1-y^2}+\dfrac{1}{1+2y+y^2} \sqcap \dfrac{1+2y\boxed{+y^2},+1\boxed{-y^2}}{1-y,,1+y,\text{cub.}}$ sive $\dfrac{2}{1-y,,1+y,\square}$.

466,14 usque ad finem paginae: Bis zur Stufe *(3) (a)* der Lesart zu S. 466 Z. 12 – Z. 1 6 [*Fig. 1*]: Leibniz hat die Strecken HL und FG getrennt gezeichnet. Dem Text gemäß müßten die Punkte H und F und damit auch L und G zusammenfallen.

omnium DE, applicatarum ad AC, erit: $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$ etc. $\sqcap$ spatio in infinitum producto, $ACBEM$.

Sed $HL \sqcap \frac{1}{1+y} \sqcap 1-y+y^2-y^3+y^4-y^5$ etc. et summa omnium HL. ad CF. applicatarum erit:

$$\underbrace{1-\frac{1}{2}}_{\frac{1}{2}}+\underbrace{\frac{1}{3}-\frac{1}{4}}_{\frac{1}{12}}+\underbrace{\frac{1}{5}-\frac{1}{6}}_{\frac{1}{30}} \text{ etc. } \sqcap \text{ spatio } CFGLB.$$

Auferatur spatium $CFGLB$, a spatio $ACBEM$, fiet:

$\frac{1}{1}\left(-\frac{1}{1}\right)+\frac{1}{2}\left(+\frac{1}{2}\right)+\frac{1}{3}\left(-\frac{1}{3}\right)+\frac{1}{4}\left(+\frac{1}{4}\right)$ etc. sive $\frac{2}{2}\ \frac{2}{4}\ \frac{2}{6}$ etc. $\sqcap 1\ \frac{1}{2}\ \frac{1}{3}$ etc. Quod satis mirabile est, et ostendit, summam ipsius seriei $1.\ \frac{1}{2}.\ \frac{1}{3}.$ etc. esse infinitam, et proinde et aream spatii $ACBGM$. quandoquidem finita $CBGF$, ei ademta, idem relinquit, seu nihil notabile adimit.

Hoc argumento concluditur, infinitum non esse totum; nec nisi fictionem, alioqui enim foret pars aequalis toti.

Si ad $1.\ \frac{1}{2}.\ \frac{1}{3}.\ \frac{1}{4}.$ etc. addas $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}$ etc. fiet $2+\frac{2}{3}+\frac{2}{5}+\frac{2}{7}$ [etc.] quod aequatur spatio $GFCAMBLG$. Huius duplo[,] sumta scilicet et opposita hyperbola, $4+\frac{4}{3}+\frac{4}{5}+\frac{4}{7}$ etc. auferatur area circuli, cuius radius CF. quae est: $4-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}+\frac{4}{9}$ etc. fiet inde differentia haec: $\frac{8}{3}+\frac{8}{7}+\frac{8}{11}$ etc. et si addatur potius illi spatio area circuli eiusdem fiet: $8+\frac{8}{5}+\frac{8}{11}$ etc.

Si diameter sit 1. circumferentia est: $\frac{4}{1}-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}$ etc. Ergo si diameter 2. seu si radius 1. erit circumferentia $\frac{8}{1}-\frac{8}{3}+\frac{8}{5}-\frac{8}{7}$ [etc.] ergo semicircumferentia, $\frac{4}{1}-\frac{4}{3}+\frac{4}{5}$ etc.

1 $+\frac{1}{5}$ etc. *(1)* Posita *(2)* $\sqcap$ spatio *L* 6 ACBEM, *(1)* restabit: $2 \frown \frac{1}{2} * 2 \frown \frac{1}{4} * 2 \frown \frac{1}{6}$, *(2)* fiet: *L* 14 ad *(1)* spatium *(2)* $1.\ \frac{1}{2}.\ \frac{1}{3}.\ \frac{1}{4}.$ etc. *L* 14–469,12 etc. *erg. Hrsg. sechsmal* 15 sumta ... hyperbola, *erg. L*

quae ducta in radium 1. dat circulum, ergo circulus $\frac{4}{1}-\frac{4}{3}+\frac{4}{5}$ etc. ergo quadrans circuli $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}$ etc. Fiet: $\underbrace{1-\frac{1}{3}}_{\frac{2}{3}}+\underbrace{\frac{1}{5}-\frac{1}{7}}_{\frac{2}{35}}+\underbrace{\frac{1}{9}-\frac{1}{11}}_{\frac{2}{99}}$.

Spatium $FCBG$ dimidietur, fiet: $\underbrace{\frac{1}{2}-\frac{1}{4}}_{\frac{2}{8}}+\underbrace{\frac{1}{6}-\frac{1}{8}}_{\frac{2}{48}}+\underbrace{\frac{1}{10}-\frac{1}{12}}_{\frac{2}{120}}$ [etc.]

$$\begin{array}{ccccccccccccccccl}
\frac{1}{3} & \frac{1}{8} & \frac{1}{15} & \frac{1}{24} & \frac{1}{35} & \frac{1}{48} & \frac{1}{63} & \frac{1}{80} & \frac{1}{99} & \frac{1}{120} & \frac{1}{143} & \frac{1}{168} & \frac{1}{195} & \text{etc.} & \sqcap & \frac{3}{4}. \\
\frac{1}{3} & \cdot & \frac{1}{15} & \cdot & \frac{1}{35} & \cdot & \frac{1}{63} & \cdot & \frac{1}{99} & \cdot & \frac{1}{143} & \cdot & \frac{1}{195} & \text{etc.} & \sqcap & \frac{1}{2}. \\
\cdot & \frac{1}{8} & \cdot & \frac{1}{24} & \cdot & \frac{1}{48} & \cdot & \frac{1}{80} & \cdot & \frac{1}{120} & \cdot & \frac{1}{168} & \cdot & \text{etc.} & \sqcap & \frac{1}{4}. \\
\frac{1}{3} & \cdot & \cdot & \cdot & \frac{1}{35} & \cdot & \cdot & \cdot & \frac{1}{99} & \cdot & \cdot & \cdot & \frac{1}{195} & \text{[etc.]} & \sqcap & \text{semiquadranti.} \\
\cdot & \frac{1}{8} & \cdot & \cdot & \cdot & \frac{1}{48} & \cdot & \cdot & \cdot & \frac{1}{120} & \cdot & \cdot & \cdot & \text{[etc.]} & \sqcap & \text{quartae parti}
\end{array}$$

spatii $FCBG$.

Ad $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+\frac{1}{9}-\frac{1}{10}$ [etc.] $\sqcap$ spatio $FCBG$

addatur $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\frac{1}{13}-\frac{1}{15}$ etc. $\sqcap$ quadranti

fiet: $2-\frac{5}{6}+\frac{8}{15}-\frac{11}{28}$ etc. sed nihil inde.

Auferatur, fiet: $*-\frac{1}{6}+\frac{2}{15}-\frac{3}{28}+\frac{4}{45}$ etc. quae series proinde pendet ex iunctis circulari et hyperbolica.

Sed omnium optima auferendi ratio haec videtur:

2 $-\frac{1}{7}$ etc. *(1)* Auferatur quadrans circuli | a spatio CFGB. *streicht Hrsg.* | *(a)* restabit *(b)* restabit *(c)* restabit: *(2)* Fiet: *L*   4 *(1)* Hyperbola *(2)* Spatium FCBG *(a)* dupletur *(b)* dimidietur *L*
11 f. FCBG. *(1)* A ... spatio FCBG auferatur *(2)* Ad *L*   15 $+\frac{4}{45}$ etc. *(1)* cui si addatur *(2)* quae *L*

Ab $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\frac{1}{13}-\frac{1}{15}+\frac{1}{17}-\frac{1}{19}$ [etc.] $\sqcap$ quadranti auferatur $\frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\frac{1}{10}-\frac{1}{12}+\frac{1}{14}-\frac{1}{16}+\frac{1}{18}-\frac{1}{20}$ [etc.] $\sqcap$ dimidio spatio $FCBG$. Restabit: $\frac{1}{2}-\frac{1}{12}+\frac{1}{30}-\frac{1}{56}+\frac{1}{90}-\frac{1}{132}$ etc.

$$\text{Iam } \underbrace{1-\frac{1}{2}}_{\frac{1}{2}}+\underbrace{\frac{1}{2}-\frac{1}{3}}_{\frac{1}{6}}+\underbrace{\frac{1}{3}-\frac{1}{4}}_{\frac{1}{12}}+\underbrace{\frac{1}{4}-\frac{1}{5}}_{\frac{1}{20}}+\underbrace{\frac{1}{5}-\frac{1}{6}}_{\frac{1}{30}}+\underbrace{\frac{1}{6}-\frac{1}{7}}_{\frac{1}{42}} \text{ etc.}$$

$$\frac{1}{56} \quad \frac{1}{72} \quad \frac{1}{90} \quad \frac{1}{110} \text{ etc.} \sqcap 1.$$

$$\text{Et } \underbrace{1-\frac{1}{2}}_{\frac{1}{2}} \quad \underbrace{\frac{1}{3}-\frac{1}{4}}_{\frac{1}{12}} \quad \underbrace{\frac{1}{5}-\frac{1}{6}}_{\frac{1}{30}} \quad \underbrace{\frac{1}{7}-\frac{1}{8}}_{\frac{1}{56}} \quad \underbrace{\frac{1}{9}-\frac{1}{10}}_{\frac{1}{90}} \text{ etc.} \sqcap$$

spatio $FCBG$.

Ergo recolligendo spat. $FCGB \sqcap \frac{1}{2} \cdot \frac{1}{12} \cdot \frac{1}{30} \cdot \frac{1}{56} \cdot \frac{1}{90}$ etc.

et quadrans $-\frac{\text{spat. } FCBG}{2} \sqcap \frac{1}{2}-\frac{1}{12}+\frac{1}{30}-\frac{1}{56}+\frac{1}{90}$ etc.

$$\text{Ergo } \frac{\text{semicirculus } -\frac{1}{2}-\frac{1}{12}-\frac{1}{30}-\frac{1}{56}-\frac{1}{90}\text{[etc.]}}{2} \sqcap \frac{1}{2}-\frac{1}{12}+\frac{1}{30}-\frac{1}{56}+\frac{1}{90} \text{ etc.}$$

$$\text{sive semicirculus} \sqcap \underbrace{\left(\frac{1}{1}+\frac{1}{2}\right)}_{\frac{3}{2}} \underbrace{\left(-\frac{1}{6}+\frac{1}{12}\right)}_{-\frac{1}{12}} \underbrace{\left(+\frac{1}{15}+\frac{1}{30}\right)}_{+\frac{3}{30}} \underbrace{\left(-\frac{1}{28}+\frac{1}{56}\right)}_{-\frac{1}{56}} \text{ etc.}$$

Unde sequitur semicirculum excedere triplam differentiam dimidii spatii hyperbolici a quadrante duplo huius seriei, $\frac{1}{12}\ \frac{1}{56}\ \frac{1}{132}$, vel semicirculum excedere differentiam quadrantis a spatio dicto, duplicata serie: $\frac{1}{2}\ \frac{1}{30}\ \frac{1}{90}$.

1–11 etc. *erg. Hrsg. dreimal* 9 spat. FCBG *(1)* $\sqcap \frac{1}{6}\ \frac{1}{20}\ \frac{1}{42}\ \frac{1}{72}\ \frac{1}{100}$ etc. *(2)* $\sqcap \frac{1}{2}$ *L*
14 semiculum *(1)* esse *(2)* aequari *(a)* triplae differentiae dimidii spatii hyperbolici et quadrantis *(aa)* demto *(bb)* si duplo huius seriei, $\frac{1}{12}\ \frac{1}{56}\ \frac{1}{132}$ sit minutum, *(b)* deficere *(c)* a tripla differentia dimidii spatii hyperbolici et quadrantis *(aa)* differre *(bb)* deficere *(3)* excedere *(4)* excedere *(5)* excedere *L*

Esto semicirc. $\sqcap S$. differentia dicta $\sqcap D$, series $\frac{1}{2}\ \frac{1}{30}\ \frac{1}{90} \sqcap L$. series $\frac{1}{12}\ \frac{1}{56}\ \frac{1}{132} \sqcap M$, erit $S \sqcap 3D+2M$. et idem $S \sqcap D+2L$. Ergo $3D+2M \sqcap D+2L$. Ergo $D \sqcap L-M$. quae calculi probatio est elegans.

Quoniam ergo semicirculus $S \sqcap 3D$+[$2M$] duplo seriei huius $\left[M\ \frac{1}{12}\ \frac{1}{56}\ \frac{1}{132}\right]$ minutus triplo differentiae D qua quadrans dimidium spatium hyp. dictum excedit, aequatur; et idem semicirculus duplo huius seriei $\left[L\ \frac{1}{2}\ \frac{1}{30}\ \frac{1}{90}\ \text{minutus}\right]$ ipsi differentiae D aequatur, $S \sqcap D$[$+2L$] ideo vel seriem L. vel seriem M. utramlibet earum separatim per circulum vel hyperbolam inveniri sufficit. Quod tamen ut clarius appareat, efficiendum est:

Series $\frac{1}{2}\ \frac{1}{12}\ \frac{1}{30}\ \frac{1}{56}\ \frac{1}{90}$ etc. habetur ex data hyperbolae quadratura, quod si iam etiam $\frac{1}{2}\ \frac{1}{30}\ \frac{1}{90}$ [etc.] habetur ex eadem, tunc etiam $\frac{1}{12}\ \frac{1}{56}\ \frac{1}{132}$ [etc.] ex quad. hyp. habebitur.

$L+M \sqcap c$. quam c. suppono cognitam ex quad. hyp. Ergo si $L \sqcap d$. cognitae etiam ex $\square$ hyp. erit $M \sqcap c-d$. et $L-M \sqcap d-c+d$. Ergo $L-M \sqcap 2d-c$. Iam $L-M \sqcap e-f$. posito e. haberi ex quad. circ. seu esse quadrantem, et f. esse dimidium spatium hyperbolicum, $FCBG$. eritque $2d-c \sqcap e-f$. Eritque e. quadrans $\sqcap 2d-c+f$. cognitis ex hyperbolae quadratura. Tantum ergo superest ut seriem L, $\frac{1}{2}\ \frac{1}{30}\ \frac{1}{90}$ etc. ex sola hyperbolae quadratura ducere tentemus. Figura autem cuius series quadratrix L, est $\frac{1}{y^3+y^2+y+1}$, sive $\frac{1}{1+y^2,\frown 1+y}$, sive $\frac{1-y}{1-y^4}$ ut initio huius plagulae dixi. Cuius figurae quadraturam ex quadratura hyperbolae superest ut deducamus si licet. Eius momentum ex $1+y$ est $\frac{1}{1+y^2}$, quae pendet ex quad. circuli; ita ut summa totius illius

4 2L *L ändert Hrsg.* 4 L $\frac{1}{2}\ \frac{1}{30}\ \frac{1}{90}$ *L ändert Hrsg.* 6 M $\frac{1}{12}\ \frac{1}{56}\ \frac{1}{132}$ auctus *L ändert Hrsg.* 7 −2M *L ändert Hrsg.* 11 etc. *erg. Hrsg. zweimal* 11 f. hyp. | pura *gestr.* | habebitur *L* 18 quadratura *(1)* ducamus *(2)* ducere *L* 19 ut . . . dixi *erg. L* 20 si licet *erg. L*

19 initio: s. o. S. 465 Z. 15 f.

figurae aequetur quadranti, e. Eius momentum ex y est: $\frac{y}{1+y^2,\frown 1+y}$. Ergo $\frac{1}{1+y^2} - \frac{y}{1+y^2,1+y} \sqcap \frac{1}{1+y^2,1+y}$. Iam: $\frac{y}{1+y^2,1+y}$ momentum ex $1+y$ habet $\frac{y}{1+y^2}$, seu $\frac{f}{2}$ spatium scilicet hyperbolicum dimidiatum; momentum ex y, habet: $\frac{y^2}{1+y^2,1+y}$. Ergo $\frac{y}{1+y^2} - \frac{y^2}{1+y^2,1+y} \sqcap \frac{y}{1+y^2,1+y}$. Iam $\frac{y^2}{1+y^2,1+y}$ ex $1+y$ habet momentum $\frac{y^2}{1+y^2}$ ex quad. circ. et ex y, habet $\frac{y^3}{1+y^2,1+y}$ et ex $y-1$ habet $\frac{y^3-y^2}{1+y^2,1-y^2} \sqcap \frac{y^3-y^2}{1-y^4}$. quae omnes proinde pendent ex circuli et hyperbolae coniunctorum quadratura; sed nihil inde ad quadraturam circuli ex hyperbola vel contra duci potest.

$\frac{a^2}{a^2+y^2} \sqcap x$. Ergo differentia duarum $x \sqcap \frac{a^2}{a^2+y^2} - \frac{a^2}{a^2+y^2+2y\beta+\beta^2} \sqcap$ $\frac{\boxed{a^2}\boxed{+y^2}+2y\beta+\beta^2,,\frown a^2,\boxed{-a^4}\boxed{-a^2y^2}}{a^4+2a^2y^2+y^4}$ fiet: $\frac{2y}{a^2+y^2,a^2+y^2} \sqcap w$, cuius figurae habetur quadratura.

Si sit: $\frac{a^3}{a^2-y^2-2y\beta-\beta^2} - \frac{a^3}{a^2-y^2}$. unde $\frac{\boxed{a^3,\frown a^2-y^2},-,,a^3,\frown\boxed{a^2-y^2}-2y\beta-\beta^2}{a+y,\square,a-y,\square}$. Haec figura ergo $\frac{\not{2}ya^4}{a+y,\square,a-y,\square}$ est qua-

4 $\sqcap \frac{y}{1+y^2,1+y}$ *(1)* vel $\frac{1}{1+y^2} - \frac{y}{1+y^2,1+y} \sqcap$ *(2)*. Iam *L* 7 f. potest. *(1)* $2ax - x^2 \sqcap y^2$, investigando intervallum tangentium $\not{2}al - \not{2}xl \sqcap \not{2}y^2$, sive $l \sqcap \frac{y^2}{2a-x}$ *(!)*, sive $l \sqcap \frac{2ax-x^2}{2a-x}$. Est autem $\frac{l}{y} \sqcap \frac{a}{w}$. Ergo $w \sqcap \frac{ya,\frown 2a-x}{2ax-x^2}$. Ergo $w \sqcap \frac{a\sqrt{2ax-x^2}\frown 2a-x}{\sqrt{2ax-x^2}\frown\sqrt{2ax-x^2}}$. Ergo $w \sqcap \frac{2a^2-xa}{\sqrt{2ax-x^2}}$. Iam *(a)* $\frac{a^2}{\sqrt{2ax-x^2}} \sqcap$ *(b)* $\frac{a^4}{2ax-x^2} \sqcap y^2$ *(aa)* $\frac{a^2}{y} \sqcap$ *(bb)* $\frac{a^4}{2ax-x}$ *(2)* $\frac{a^2}{a^2+y^2}$ *L* 11 sit: *(1)* $\frac{a^2}{a^2-y^2} - \frac{a^2}{a^2-y^2-2y\beta-\beta^2} \sqcap w$. Unde *(a)* $\overline{a^2,\frown\boxed{a^2-y^2}-2y\beta\boxed{-\beta^2},,\boxed{-a^2,\frown a^2\boxed{-y^2}}}$ *(b)* $\overline{a^3,\frown\boxed{a^2-y^2}-2y\beta\boxed{-\beta^2},,\boxed{-a^3,\frown a^2\boxed{-y^2}}}$ *(2)* $\frac{a^3}{a^2-y^2-2y\beta-\beta^2}$ *L*

drabilis. Eius momentum ex $a+y$, est $\frac{\not 2 y}{a+y,a-y,\square}$. Eius momentum ex $a-y$, est $\frac{\not 2 y}{a^2-y^2,a+y}$.

$\frac{y}{1+y,,1-y,\square}+\frac{y}{1+y,1-y^2}$ aequatur $\frac{2y}{1+y,1-y,,\square}$. ideoque haec summa quadrabilis. Investigemus iam: $\frac{y}{a+y,,a-y,\square}$. Eius momentum ex $a+y$, est $\frac{y}{a-y,\square}$, eius momentum ex $a-y$, est $\frac{y}{a^2-y^2}$.

$\frac{y}{1-y,\square}+\frac{y}{1-y^2}\sqcap\frac{2y}{1+y,,1-y,\square}$. Investigemus: $\frac{y}{1-y,\square}$. addatur $\frac{1}{1-y,\square}$ quae habetur absolute. $\frac{1-y}{1-y,\square}\sqcap\frac{1}{1-y}$. quae pendet ex quad. hyp. Ergo $\frac{y}{1-y,\square}$ pendet ex quad. hyp. Investigemus et $\frac{y}{1-y^2}$, [eius] momentum ex $1+y$, est $\frac{y}{1-y}$, quae pendet ex quad. hyp. Eiusdem momentum ex $1-y$, est $\frac{y}{1+y}$, quod etiam pendet ex quad. hyp. Ergo habetur et tertium momentum $\frac{y^2}{1-y^2}$, ergo et $\frac{1}{1-y^2}$ ex ead. quad. hyp.

$\frac{1+y^2+y^4+y^6}{1\quad\frac{1}{3}\quad\frac{1}{5}\quad\frac{1}{7}}$. Ergo $1+\frac{1}{5}+\frac{1}{9}+\frac{1}{13}$ [etc.], vel $\frac{1}{3}+\frac{1}{7}+\frac{1}{11}$ etc. pendent ex quad. circ. et hyp. simul: Ergo $\frac{1}{1-y^4}$ pendet ex quad. circ. et hyperb. simul. Hoc obiter. Ergo $\frac{y}{1+y,,1-y,\square}$ pendet ex $\square$ hyp.: Ergo et $\frac{y}{a^2-y^2,a+y}$ pendet ex quad. hyp. Quod an

1 a + y, est *(1)* latus *(2)* $\frac{\not 2 y}{a+y,a-y,\square}$ *L* 2 f. $\frac{\not 2 y}{a^2-y^2,a+y}$. *(1)* Differentia ergo inter $\frac{y}{a+y,,a-y,\square}-\frac{y}{a+y,a^2-y^2}$ aequatur *(a)* cylindro fi *(b)* $\frac{\not 2 y}{a+y,a-y,,\square}$ *(2)* $\frac{y}{1+y,,1-y,\square}$ *L* 6 Investigemus: $\frac{y}{1-y,\square}$. *(1)* Eius *(2)* Haec pendet ex quad. hyp. *(3)* addatur *L* 8 ex *L ändert Hrsg.* 11 etc. *erg. Hrsg.*

472,11–473,2 Leibniz verwendet punktuell die Homogenitätsfaktoren a^3 bzw. a^4 und streicht dreimal den für die Aussage belanglosen Faktor 2.

aliunde pateat videamus: eius momentum ex $a-y$, est $\frac{y}{a+y,\Box}$. quod habetur absolute. Eius mom. ex $a+y$, est $\frac{y}{a^2-y^2}$. quod etiam habetur ex quad. hyp. Sed satis de his. Sequitur plagula IX.

38_{11}. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS NONA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 20–21. 1 Bog. 2°. 4 S.
Cc 2, Nr. 775 A tlw.

Schediasmatis de seriebus et quadraturis pars IX.

Quoniam sub finem praecedentis plagulae ostensum est differentias ipsarum $\frac{a^2}{a^2+y^2}$ esse $\frac{2y}{a^2+y^2,\square}$. ductis ergo ipsis in distantias a vertice, fiet: $\frac{2y^2}{a^2+y^2,\square}$, quae figura aequatur ipsi figurae $\frac{a^2}{a^2+y^2}$ et proinde pendet ex quad. circuli. Quodsi ei addatur $\frac{a^2}{a^2+y^2,\square}$, momentum ipsarum $\frac{a^2}{a^2+y^2}$ ex asymptoto, quod ideo etiam ex quad. circuli pendet, fiet: $\frac{a^2+y^2}{a^2+y^2,\square}$, sive $\frac{1}{a^2+y^2}$, quae pendet ex quad. circuli. Ergo differentia ipsarum $\frac{a^2}{a^2-y^2}$[,] $\sqcap$ $\frac{2y}{a^2-y^2,\square}$ $\sqcap$ $\frac{2y}{a+y,a-y,\square}$ quae figura quadrari potest. Ergo $\frac{\not{2}y}{a+y,\square,a-y}+\frac{\not{2}y}{a-y,\square,a+y}$ quadrari potest. Item: $\frac{y^2}{a+y,a-y,\square}+\frac{y}{a+y,\square,a-y}$, quadrari potest. Item $\frac{y}{a-y,\square,a+y}-\frac{y^2}{a+y,a-y,\square}$.

Subit animum nova quaedam materia inquirendi, e.g. ope huiusmodi calculorum quadrari possunt figurae quaedam non geometricae, e.g. hyperbola est $\frac{1}{1\mp y}$, eius momentum haberi potest. Haberi ergo et potest dimensio figurae, quae sit quadratrix hyperbolae.

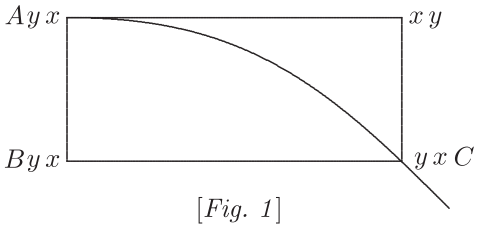
6 a vertice *erg.* *L* 7 aequatur *(1)* complemento figurae *(2)* ipsi *L* 8 $\frac{\mathrm{a}^2}{\mathrm{a}^2+\mathrm{y}^2}$ ex *(1)* basi, *(2)* asymptoto *L* 12 f. $-\frac{\mathrm{y}^2}{\mathrm{a+y,a-y},\square}$. *(1)* Ope huius meth *(2)* In *(3)* Subit *L* 13 e.g. *(1)* differentiae *(2)* ope *L* 14 e.g. *(1)* sit hyperbola *(2)* circulus est *(3)* hyperbola *L*

11 Leibniz streicht den für die Aussage belanglosen Faktor 2 im Zähler der beiden Summanden.

Haec figura etsi non sit geometrica, tamen poterit forte describi eo modo quo cycloeides; et volutae, et helices; et ita eiusmodi figurarum non geometricarum, quadratura absoluta saepe haberi hac methodo poterit. Ita, circulus $\sqrt{a^2 - x^2}$, multiplicetur per $a + x$, sive per $\sqrt{a + x, \square}$ fiet: $\sqrt{a + x, \text{cub.}, a - x}$ cuius figurae dimensio dabit dimensionem figurae circuli quadratricis. Itaque videndum an generaliter hoc solvi possit: Propositis figuris non geometricis, geometricas symmetros invenire. Quod foret maximi momenti problema generale.

$\frac{y}{a} - \frac{a}{y} \sqcap \frac{y^2 - a^2}{ay}$, quae figura est quadrabilis. Differentia ordinatarum $\frac{y^2 + 2\beta y + \beta^2 - a^2}{ay + a\beta} - \frac{y^2 - a^2}{ay} \sqcap \frac{ay^3 + 2a\beta y^2 + a\beta^2 y - a^3 y, -ay^3 - a\beta y^2 + a^3 y + a^3\beta}{a^2y^2 + a^2y\beta}$, sive: $\frac{a\not\beta y^2 + a^3\not\beta}{a^2y^2}$, quae est quadrabilis.

Aliter:



[*Fig. 1*]

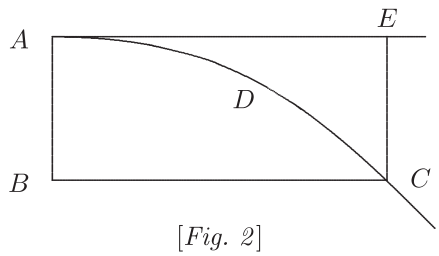
Exposita esto figura, cuius aequatio sit $\frac{ya^2}{a^2 + y^2} \sqcap x$. Momentum ipsarum ordinatarum huius figurae ex y, est $\frac{y^2 a}{a^2 + y^2}$. Quaeritur valor ipsarum y. ex aequatione $\frac{ya^2}{a^2 + y^2} \sqcap x$, fiet: $a^2x + xy^2 \sqcap ya^2$. Unde $y^2 - \frac{a^2}{x}y + \frac{a^4}{x^2} \sqcap \frac{a^4}{x^2} - a^2$. sive $\mp y \pm \frac{a^2}{x} \sqcap \frac{a\sqrt{a^{\not 4 2} - x^2 \not a^{\not 2}}}{x}$. et

5 Itaque *(1)* propositis lineis *(2)* videndum *L* 10 f. quadrabilis. *(1)* | Hinc *streicht Hrsg.* | *(2)* Ascribere operae pretium est modum, quo demonstrari potest figurae $\frac{\mathrm{a}^2}{\mathrm{a}^2 + \mathrm{y}^2}$ dimensio pendens a circulari: *(3)* Aliter *L*

15–477,2 Unde: Bei der quadratischen Ergänzung addiert Leibniz $\frac{a^4}{x^2}$ statt $\frac{a^4}{4x^2}$ zu beiden Seiten der Gleichung und verfehlt so die richtige Lösung. Der Fehler beeinträchtigt die Rechnung bis S. 477 Z. 2. Analoge Fehler treten in den S. 477 Z. 10 u. S. 478 Z. 5 auf und wirken sich bis S. 479 Z. 3 aus.

$y \sqcap \dfrac{\mp a\sqrt{a^2-x^2}+a^2}{x}$. Quadrata omnium y, seu momentum complementi figurae ABC, dat: $\dfrac{a^4-a^2x^2+a^4 \mp 2a^3\sqrt{a^2-x^2}}{x^2}$ quae figura pendet ex q. circuli.

Resumemus:



[*Fig. 2*]

Esto curva ADC. $AB \sqcap x \sqcap EC$. $BC \sqcap y \sqcap AE$. Esto aequatio: $a^2y \sqcap a^2x+y^2x$. Erit $x \sqcap \dfrac{a^2y}{a^2+y^2}$ quae pendet a quad. hyp., et $x^2 \sqcap \dfrac{a^4y^2}{a^2+y^2, \square}$, quorum summa est momentum figurae $AECDA$ ex axe aequilibrii AE. At summa omnium $\dfrac{a^2y^2}{a^2+y^2}$, quae pendet ex quad. circuli est momentum figurae $AECDA$ ex axe aequilibrii AB.

Iam ut quaeramus y, ita procedendum est, ex aequatione figurae fiet: $y^2x-a^2y \sqcap -a^2x$, sive $y^2-\dfrac{a^2}{x} \sqcap -a^2$, sive $y^2-\dfrac{a^2}{x}y+\dfrac{a^4}{x^2} \sqcap \dfrac{a^4-a^2x^2}{x^2}$, unde $\mp y \pm \dfrac{a^2}{x} \sqcap \dfrac{a\sqrt{a^2-x^2}}{x}$. [Unde patet ut obiter dicam figuram $\dfrac{a^2\sqrt{a^2-x^2}}{x}$, pendere a quad. hyperbolae. Nam ipsae y pendent a quad. hyp. aufer inde ipsas $\dfrac{a^2}{x}$ quae pendent a quad. hyp. etiam

1 $\mathrm{y} \sqcap \dfrac{\mp \mathrm{a}\sqrt{\mathrm{a}^2-\mathrm{x}^2}+\mathrm{a}^2}{\mathrm{x}}$. *(1)* Momentum *(2)* et momentum omnium x, *(3)* Quadrata *L*
1 f. complementi *(1)* omnium *(2)* figurae *(a)* y, yx, yx *(b)* ABC, *(aa)* est *(bb)* dat *L* 6 quae ... hyp. *erg. L* 7 f. aequilibrii *erg. L zweimal* 7 f. quae ... circuli *erg. L*

10+478,5 $+\dfrac{a^4}{x^2}$ u. $+\dfrac{y^4}{a^2}$: s. o. Erl. zu S. 476 Z. 15 – Z. 2. 11+478,9 Die eckigen Klammern stammen von Leibniz.

$\dfrac{a\sqrt{a^2-x^2}}{x}$ pendebit a quad. hyp. Eius vero centrum gravitatis pendet a quad. circuli, nam si ducatur in x, fiet: $a\sqrt{a^2-y^2}$, quae est homogenea circulo, si ducatur in se, fiet: $\dfrac{a^4-a^2x^2}{x^2}$, quae habetur absolute: Habito ergo figurae centro gravitatis ex data circuli quadratura, quaeramus alteram eius indeterminatam, fiet: $a^4-a^2x^2 \sqcap y^2x$, unde ordinando $x^2+\dfrac{y^2}{a^2}x+\dfrac{y^4}{a^2} \sqcap \dfrac{y^4+a^4}{a^2}$. Unde $x+\dfrac{y^2}{a} \sqcap \dfrac{\sqrt{y^4+a^4}}{a}$ ((quae figura patebit ex quad. hyperbolae. Ergo et $\dfrac{\sqrt{y^4+a^4}}{a}$, pendebit ex quad. hyp. Ergo $a^4 \sqcap a^2x^2-y^4$ ex quad. hyp. sive $a^4 \sqcap ax-y^2, \frown ax+y^2$. Unde $a\sqrt{a^2-x^2} \sqcap y^2$. momentum ergo omnium y ex axe eorum ex quad. circuli. Sed $\sqrt{a\sqrt{a^2-x^2}} \sqcap y$. quae figura pendet ex q u a d. h y p e r b o l a e. Sed $x\sqrt{a\sqrt{a^2-x^2}}$ pendet ex q u a d r a t u r a c i r c u l i .))]
Invenimus ut ad figuram nostram redeamus in ea momentum figurae $AECDA$ ex AB pendere ex quad. circuli quia $xy \sqcap \dfrac{a^2y^2}{a^2+y^2}$. Momentum vero figurae [$ABCDA$], ex AE, pendet etiam ex quadratura circuli, quia $xy \sqcap \ddagger a\sqrt{\sqrt{a^2-x^2}}+a^2$, quia inventa est $y \sqcap \dfrac{\ddagger a\sqrt{a^2-x^2}+a^2}{x}$. Ideoque centrum gravitatis figurae propositae pendet ex quad. circuli; ergo $\dfrac{a^4y^2}{a^2+y^2,\square}$, item $\dfrac{a^4-a^2x^2 \ddagger 2a^3\sqrt{a^2-x^2}+a^4}{x^2}$, et per consequens $\dfrac{a^2\sqrt{a^2-x^2}}{x^2}$ ex quad. circuli pendebit.

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6 *Daneben:* Vide plag. 10. pag. ult.

8 f. *Dazu am Rande:* Contra potius ₰.

7 $\sqcap$ y^2. *(1)* cuius ordinata ex qu *(2)* momentum *L* 10 ea *(1)* BC, esse *(2)* momentum figurae *(a)* ex *(aa)* AE *(bb)* AB *(b)* AECDA ex *(aa)* AE *(bb)* AB *L* 11 ex ABCDE *L ändert Hrsg.*

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16 Vide: s. u. N. 38$_{12}$ S. 495 Z. 3–5.

Huius figurae [quadrati ordinatarum] y^2, habentur absolute, sed y, ex x, dant $\dfrac{a^2\sqrt{a^2-x^2}}{x} \sqcap y$. Unde $a^6 - a^4x^2 \sqcap y^2x^4$. Unde $x^4 + \dfrac{a^4x^2}{y^2} + \dfrac{a^8}{y^4} \sqcap \dfrac{a^8}{y^4} + \dfrac{a^6}{y^2}$ et $x^2 + \dfrac{a^4}{y^2} \sqcap \dfrac{\sqrt{a^8+a^6y^2}}{y^2}$, et $x \sqcap \dfrac{\sqrt{\sqrt{a^8+a^6y^2}-a^4}}{y}$.

Data figura curvam ei homogeneam invenire, elegantissimum est, atque utilissimum problema, ita enim ope curvae huius evolutae, secari potest figura proposita, in ratione data, et cuilibet eius portioni dari rectilineum aequale. Methodus autem id quaerendi haec est:

A quadrato ordinatae figurae datae auferatur quadratum unitatis; quodsi iam residui radix est ordinata figurae quadrabilis, satisfieri potest postulato.

Ut esto figura circulus: eius ordinata $\sqrt{2ax-x^2}$, eius quadratum $2ax-x^2$, auferatur inde a^2, fiet: $\sqrt{2ax-x^2-a^2}$. Unde patet ad hanc curvam habendam ipsius circuli opus esse quadratura. Sed quodsi eius quadratum sit minus quadrato unitatis id utique inde auferri non poterit, et tunc nulla dabitur curva illis homogenea, eo quidem ordinatarum sensu, ut in circulo, ab a^2-y^2, si auferas a^2 restabit $-y^2$, cuius radix est impossibilis.

In hyperbola, sit a^2+y^2, aufer inde a^2 fiet y^2, cuius radix y. Unde sequeretur figuram ipsarum y quadratricem seu omnium $\dfrac{y^2}{2}$ quae est parabola cuius aequatio: $y^2 \sqcap 2ax$. habere curvam homogeneam figurae hyperbolae. Hinc si aliter assumas ordinatas figurae hy-

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14 *Dazu am Rande:* Imo ab a^2-y^2, auferri debet b^2, et res procedet.

478,15–479,1 pendebit. *(1)* Eius momentum *(2)* Huius figurae | quadrati ordinatarum *gestr. erg. Hrsg.* | y^2 *L* 3 f. $x \sqcap \dfrac{\sqrt{\sqrt{a^8+a^6y^2}-a^4}}{y}$. *(1)* Sed ista exitum *(2)* Data *L* 5 ope *(1)* fili *(2)* curvae *L* 7 f. est: *(1)* A *(2)* Datae figur *(3)* Quadratum ordinatae figurae datae aufe *(4)* A *L* 9 f. postulato *(1)*, ex notis *(2)*. Ut *L* 12 quadratura. *(1)* | Aliter, *streicht Hrsg.* | *(a)* $a^2-y^2 \sqcap$ *(b)* $2a^2$ *(c)* a^2-y^2-a *(2)* Sed *L* 14 cuius *(1)* ordinatae *(2)* radix *L* 14 f. impossibilis. *(1)* In hyp *(2)* In parabola: *(3)* In *L* 16 $\dfrac{y^2}{2}$ *(1)* | esse *streicht Hrsg.* | hyperbola *(2)* quae *L* 16 f. $y^2 \sqcap 2ax$. *(1)* esse homogeneam curvae *(2)* habere *L*

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2 $\sqcap y^2x^4$: Richtig wäre y^2x^2. Der Fehler und die falsche quadratische Ergänzung beeinträchtigen das Ergebnis in Z. 3.

perbolicae, figura ipsis homogenea semper eveniri poterit, quia enim semel inveniri potuit, invenietur semper, prorsus quemadmodum centro gravitatis ab uno latere reperto, datur id ex aliis omnibus. Esto ergo aequatio $2ax + x^2$, inde auferatur a^2, fiet: $\sqrt{2ax + x^2 - a^2}$ cuius figurae quaeritur summa, quae summa est quadratrix ipsius hyperbolae. Itaque iam video applicatis hyperbolae ad axem, ad abscissas ex vertice; homogeneam curvam non posse haberi nisi supposita ipsa hyperbolae quadratura, nam pro x ponendo $z - a$, fiet: $\sqrt{\boxed{2az} - 2a^2 + z^2 \boxed{-2az} \boxed{+a^2 - a^2}}$, unde fiet $\sqrt{z^2 - 2a^2}$. Ad asymptoton hoc modo: $\dfrac{a^4}{x^2} - a^2$. fiet: $\sqrt{\dfrac{a^4 - a^2x^2}{x^2}}$ sive $\dfrac{a}{x}\sqrt{a^2 - x^2}$ quae pendet a quadratura hyperbolae; ut iam quaeratur figura homogenea circulo: ab $a^2 - y^2$, auferatur alia recta constans: b^2, fiet: $\sqrt{a^2 - b^2 - y^2}$ quae rursus est ad circulum.

Ab anonymae $\dfrac{ax}{\sqrt{2ax - x^2}}$ quadrato, auferatur a^2, fiet: $\dfrac{a^2x^2}{2ax - x^2}$ seu $\dfrac{a^2x}{2a - x}$, unde auferatur a^2, fiet: $\dfrac{a^2x[-]2a^3 + a^2x}{}$, sive $\sqrt{\dfrac{[-]2a^3 + 2a^2x}{2a - x}}$ figura ergo $\dfrac{[-]2a^3 + 2a^2x}{2a - x} \sqcap y^2$, sed si iussissem auferri: $a\gamma$. fiet: $\sqrt{\dfrac{a^2x - 2a^2\gamma + xa\gamma}{2a - x}}$ et γ. ita explicanda iam est ut numerator dividi possit per $2a - x$, nempe

$$\begin{array}{l} +\quad a^2x - 2a^2\gamma \not f - a^2 - a\gamma \\ \underline{+\ a\gamma \qquad\qquad\qquad} \\ \quad -\ x + 2a \end{array}$$

. Et fiet: $-2a^3 - 2a^2\gamma \sqcap -2a^2\gamma$ quod est impossibile, non ergo procurari potest divisibilitas. Saepe vero poterit hac arte procurari.

Iam si rursus a $\sqrt{\dfrac{a^2x - 2a^2\gamma + xa\gamma}{2a - x}}$ quadrato, auferas alterius constantis v. g. $\sqrt{a\delta}$, quadratum nempe $a\delta$, fiet alia iterum figura.

Nimirum verum est, figurae ex hac quadrati vel rectanguli constantis ablatione [factae] radicum, summam dare curvam syntomon figurae datae, sed non inde patet, hanc

4 quae *(1)* figura *(2)* summa *L* 6 quadratura *(1)* | sed *streicht Hrsg.* | hanc *(2)*. Imo potest explicando x *(3)*, nam *L* 9 circulo: *(1)* sinus circuli sunt *(2)* ab *L* 12 + *L ändert Hrsg.* 12 − *erg. Hrsg. zweimal* 12 $\sqcap$ y², *(1)* pendebit ex quad. circuli *(2)* sed *L* 19 f. figura. *(1)* Sed video me lapsum *(2)* Nimirum *L* 20 est, *(1)* | a *streicht Hrsg.* | figurae ex hac quadrati vel rectanguli constantis | ablatione facti *ändert Hrsg.* | quadratura pendere radicum summa *(2)* figurae *L* 21 patet, *(1)* figuram *(2)* hanc *L*

summam a quadratura figurae pendere cui syntomos quaeritur. Notabile autem est, quadratum constans, quod aufertur esse arbitrarium; et proinde aliquando tale sumi posse, ut figura inde proveniens sit alia atque alia, v. g. si sit $a^2 - x^2 + bx \sqcap y^2$, quae est ad circulum, ut ei quaeratur homogenea, fiet: $\sqrt{a^2 - x^2 + bx - a\gamma}$. cuius figurae quadratura habetur curva elementis circuli homogenea. Sed haec figura est modo circulus modo hyperbola. Imo erro: est semper circulus.

Interim in alio casu fieri potest ut alia ita fiat figura, v. g. sit: $\sqrt{b^2 - 2ax + x^2}$, quae est ad hyperbolam, eius ordinatis quaeratur homogenea, fiet $\sqrt{b^2 - 2ax + x^2 - a\gamma}$, pone $b^2 - a\gamma \sqcap a^2$, fiet $\sqrt{a^2 - 2ax + x^2} \sqcap \mp a \pm x$. ac proinde haberi potest homogenea hyperbolae.

Idem in infinitis aliis fieri poterit: tum radicum extractiones tum divisiones reddendo possibiles,: v. g. sit curva cuius aequatio: $\dfrac{b^2 - x^2}{a - x} \sqcap y^2$. Ab y^2, auferatur γa, fiet: $\dfrac{b^2 - x^2 - \gamma a^2 + \gamma a x}{a - x}$. Iamque ita licebit explicare arbitrarias, ut procedat divisio.

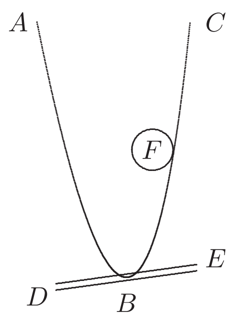
Haec valde notabilia sunt, et plane nova.

Duae figurae syntomi eiusdem curvae sunt syntomi inter se; duae curvae syntomi eiusdem figurae, sed diverso modo sumtae non sunt quidem syntomi inter se, attamen sunt symmetri, seu data dimensione unius datur et dimensio alterius.

Hinc multa nova ac plaeclara de symmetriis duci possunt. Data qualibet figura potest describi semigeometrice curva ei homogenea. Quem in finem notandum est trochoeidem cuiuslibet figurae esse homogeneam quadratrici eius figurae, quae sit curvae eius homogenea. Exempli causa: trochoeides parabolae est homogenea quadratrici hyperbolae ac proinde descripta trochoeide parabolae habebitur constructio logarithmorum geometrica et sectio rationis in data ratione. Ut autem haec trochoeides describatur necesse est laminam parabolicam sive zonam ex cono scilicet sectam arcte immissam inter duos asseres coniugatos, ut non nisi vi ibi moveri possit, ac ne recta quidem propelli, neque attolli,

5 habetur *(1)* figura circulo homogenea. Atqui huius figurae quadratura *(2)* curva *L* 5 Sed *(1)* haec *(2)* huius figurae quadratura est *(3)* haec *L* 9–11 homogenea *(1)* cir *(2)* hyperbolae. *(a)* Hinc manifest *(b)* Ponamus *(c)* Er *(d)* Idem *L* 13 ita *(1)* assumere *(2)* licebit *L* 14 f. nova. *(1)* Esto nostra ad circulum: *(a)* $\dfrac{\mathrm{x}^2}{\mathrm{a}^2 + \mathrm{y}}$ *(b)* $\dfrac{\mathrm{y}^2}{\mathrm{b}^2 + \mathrm{y}^2}$, eius quadratum $\dfrac{\mathrm{y}^4}{\mathrm{b}^4 + 2\mathrm{b}^2\mathrm{y}^2 + \mathrm{y}^4}$. *(2)* Duae *L* 17 dimensione *(1)* circuli *(2)* unius *L* 19 ei *(1)* syntomos *(2)* homogenea *L* 20 f. eius *(1)* symmetros *(2)* homogenea *L*

sed concava in parte innitendo, provolvi. Quem in finem utile erit maximi ponderis esse zonam vel suapte natura vel adiectione. Intus erit globus, et ipse gravis quantum satis est. Intus inquam, id est in concavo zonae.



[*Fig. 3*]

Zona parabolica est ABC, duo asseres sibi pene iuncti relicto pro zona nonnihil commissura, intrante interstitio, erunt DBE, inclinentur asseres ad horizontem; et globus F provolutus inclinabit zonam parabolicam quae interim stylo in ipsa defixo describet in pariete trochoidem. Sumenda est parabola plurimum differens a recta, et inprimis portio eius maxime curva. Idem est si asseres sunt horizonti paralleli, et trochoeides tum ista coarctatione tum pondere maximo imposito, non nisi mechanica ope vectis moveri possit, sed tunc aliud malum. Quod scilicet saepe circa punctum fixum rotabitur, nec progredietur. Eadem curva NB. describetur etiam si zona ponatur immobilis, sed DE circa ipsam moveantur nunc cum pariete ipsis asseribus DE affixo, sed tunc idem malum quod aliquando regredietur, et tunc non asseres sed ipsius zonae convexum habebit incisuram in qua vectis DE moveatur. Sed et iam video obiectionem ne ipsius curvae portio aliqua per idem punctum plani decurrat; non videri forte, ne quidem si zona moveatur, cum nullum sit centrum circa quod. Posteriore modo non potest fieri ut idem punctum vectis seu plani respondeat diversis curvae. Priore modo hoc quidem evenire potest. Sed non calcemus imo forte utrumque. Sed iam remedium tale satis elegans. Electo posteriore modo, hoc reddito: vectis B tendens suo pondere a C, versus E, nihilominus, manu in E premente D versus C, recolligat in se et in lineam rectam chordam ipsi BC curvae

14 aliquando *(1)* pro *(2)* regredietur *L* 16 decurrat; *(1)* cessare *(2)* non *L* 21 premente *(1)* sursum quodammodo agatur *(2)* per ipsa *(3)* D versus *L*

circumplicatam. Interea ipse pondere suo perpetuo filum tendet nec regressus ei possibilis ob filum tenens, nec progressus ob pondus renitens.

Eadem methodo et cycloeides optime describetur, paries vero vel tabula in qua designanda est curva ipsi BE affixa cum eo movebitur, stylus vero erit immobilis. Ausim dicere hanc descriptionem non fore multo difficiliorem quam parabolae secundi generis descriptionem a Cartesio propositam ad construenda problemata sursolida. Filum ipsi B infixum. Incisura in vecte, instar fossae, nisus arctioris. Duae acies eius duae acies fossae vel latitudinem incisurae ipsius zonae implebunt vel si mavis duas in ea incisuras parvas parallelas.

Vid. plag. X.

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6 Cartesio: *Geometria*, 1659, *DGS* I S. 67 f. (vgl. *DO* VI S. 442–444).

38_{12}. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS DECIMA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 22–23. 1 Bog. 2°. 4 S. Überschrift erg. Fig. 1 u. 2 stark überarbeitet.
Cc 2, Nr. 775 A tlw.

Schediasmatis de summis serierum et quadraturis figurarum pars X.

Ope huius curvae inveniri poterunt, quotcunque mediae proportionales: Imo quod est longe maius, et quod alioquin geometrice fieri non potest: Si id omne geometricum est, quod exactum est, fatendum est has constructiones fore exactas: nam linea ista tam exacte describi poterit, quam parabola secundi generis a Cartesio proposita. In quo longe differt a spiralibus quas exacte non describas, ob duos motus, a se invicem independentes, alterum in circulo, alterum in radio, quorum certa debet proportio, quam dare non est in nostra potestate, adeo ut spirales illae non nisi divina arte describi possint, ope intelligentiae, cuius cogitationes distinctae fiant in tempore minore quolibet dato; quod nec de angelis verum puto.

Quod secus est in linea nostra: Eadem linea continuo describitur motu; non per puncta invenitur.

P. Pardies, ingeniosissimus utique vir, tractatum de linea logarithmica moliebatur, descripta per medias proportionales ad rectam aequalibus intervallis applicatis; sed praeterquam quod non nisi per puncta, ac proinde non tota exacte linea describatur; intermediis punctis vacuis futuris, praeter hoc inquam quis non videt has mediarum proportionalium inventiones, aliunde esse sumendas; idque vel qualitercunque, quod non est

10 differt *(1)* ab helicibus *(2)* a spiralibus *L* 20 videt *(1)* quantae *(2)* has *L*

6 Ope huius curvae: Diese Worte befinden sich auf dem vorhergehenden Bogen, Bl. 21 v°. Leibniz hat zunächst fortlaufend geschrieben und den Bogen Bl. 22–23 erst nachträglich mit einer eigenen Überschrift versehen. 9 Cartesio: *Geometria*, 1659, *DGS* I S. 36–38 (vgl. *DO* VI S. 407–411); Leibniz setzt sich mit den daran anschließenden Bemerkungen von Descartes über die in der Geometrie zulässigen Kurven auseinander; sein Handexemplar weist auf S. 39 eine Unterstreichung auf. 17 Pardies: Die Abhandlung über die logarithmische Kurve wurde wohl nicht vollendet; vgl. *LSB* III,1 N. 9 S. 43 Erl. zu Z. 1–6; Pardies hat die logarithmische Kurve auch in den *Élémens de geométrie*, 1671 u. ö., livre VIII, articles 16–31 S. 86–93 behandelt.

geometrice sive exacte agere; vel per totidem operationes quot puncta in curva designare vult; et ille tamen ab ea linea plurimum usus publico pollicebatur.

Ecce ergo lineam geometricam quae id omne praestat quod a logarithmica exacte si possibile esset, descripta posset expectari.

Fateor si ferenda est definitio geometricarum quam dedit nobis Cartesius, nostra talis non erit: sed quemadmodum ille veteres iure culpat, quod a geometricarum numero conicas, aut certe quos vocabant lineares locos, exclusissent; ita; ille rursus culpandus est, quod geometricarum nomine ad analyticas coarctato; scientiam auxilio necessario privat; causam credo habens unicam in arcanis, ut scilicet iactare posset, omnium curvarum geometricarum a se methodum tangentesque traditas. Sane quo ille argumento utitur in veteres, eo ego in illum: quicquid exactum, id inquit geometricum est. Recte. Velim ergo ostendat, in quo descriptio evolutae circularis, vel trochoeidum nostrarum exacta non sit. Sane tam exacte descriptas esse certum est quam ullam geometricam aeque compositam. Culpat veteres, quod agnita helicum et spiralium imperfectione conchoeides quoque et cissoeides cum ipsis confusas exclusissent. Ita ego illum, quod cum helicibus et spiralibus, trochoeides et evolutas confudit. Obicit lineis quas non geometricas vocat, quod a duobus diversis motibus dependeant, quorum proportionem exacte servare non liceat. Recte illud quidem in helices vel spirales: at non recte dicitur in trochoeides nostra methodo descriptas et evolutas. Obicies; ergo habetur circuli quadratura. Nego haberi qualis quaeritur. At inquies ope trochoeidis circularis vel evolutae exacte habetur recta circumferentiae aequalis. Quis neget. Si recte descriptae, habetur ergo et geometrice. Ita certe: nam si exacte, certe geometrice. Ergo habetur et quadratum circulo aequale. Fateor. Ergo quadratura circuli. Fateor, sed non qualis quaeritur; quaeritur enim non geometrica tantum, sed et analytica: id est quaeritur ratio circuli ad quadratum inscriptum, quaeritur ratio diametri ad circumferentiam.

1 totidem *(1)* instrumenta separata, quot sunt *(2)* num *(3)* operationes *L* 7 certe *(1)* sursolidas *(2)* quas *L* 8 f. privat; *(1)* ideo *(2)* ex uno credo causam haben *(3)* causam *L* 14 quod *(1)* cognita *(2)* agnita *L* 20 ope *(1)* cycloeidis vel tro *(2)* trochoeidis circularis | vel evolutae *erg.* | exacte *L* 21 si ... descriptae *erg. L*

5–16 definitio: *Geometria*, 1659, *DGS* I S. 20 f. (vgl. *DO* VI S. 392 f.); culpat: *a. a. O.* S. 17–19 (vgl. *DO* VI S. 388–390) u. *Lettres*, Bd. 3, 1667, S. 354 (*DO* II S. 313); Obicit: *Geometria*, 1659, *DGS* I S. 18 f. (vgl. *DO* VI S. 390).

Eodem argumento, quo ille utitur in nos, veterum sectatores, et Vieta utebantur in sententiam Cartesii, usi ut Cartesiani quadratura circuli; ita illi duplicatione cubi. Nam Vieta; si lineae conicae in geometriam recipiendae sunt; habemus ergo iam duplicationem cubi, geometricam. Ita est; et duas medias proportionales exacte, fateor; ergo et hoc geometrice; ita sane oportet ex datis. Quid ergo cur quaesivere geometrae quod habebant. Facilis responsio est; quemadmodum magnitudinem circuli non tantum geometrice, sed et analytice; ita duplicationem cubi non tantum analytice, sed et plane per lineas scilicet simplicissimas rectam et circulum desiderabant. Recte ergo Cartesius lineas alias non negligendas monet ob praeclaras opticas, staticas, aliasque proprietates. Ita ego eas quae analyticae non sunt, non minus geometricas esse contendo, non tantum quod exactae sint, sed et quod praeclaros habeant usus; ut cycloeides primaria inventos ab Hugenio, secundaria observatos a Romero; trochoeides parabolica sive curva mesolaba, detectos a me.

Ita ergo distinguendum censeo; lineae omnes aut geometricae sunt aut mechanicae. Geometricae sunt, quae uno tractu descriptae intelliguntur, mechanicae quae per puncta. Unde patet lineas geometricas posse describi per puncta ut ellipsin, et contra lineas mechanicas posse describi geometrice a natura rerum; aut intelligentia directrice. Nam omnes lineae quae modo certa regula constent; omnia eorum puncta concernente, geometricae sunt, natura sui, aut ratione rerum autoris; potest etiam fieri, ut quae lineae nobis geometricae non sunt, ut logarithmica fiant aliquando, reperta eas describendi ratione; quemadmodum et evenire potest, ut lineae non analyticae fiant analyticae, quem-

5 cur | veteres *gestr.* | quaesivere *L* 9 praeclaras *(1)* in *(a)* conicis *(b)* opticis, in mechanicis, ut *(2)* opticas, | mechanicas, *gestr.* | staticas, *(a)* geom *(b)* aliasque *L* 11 primaria *erg. L* 16 puncta. *(1)* Uno tractu descriptae sunt aut *(a)* arte *(b)* in nostra potestate, aut tantum in contemplatione *(2)* Unde *L* 16 lineas | sua natura *gestr.* | geometricas *L* 17 lineas | sua natura *gestr.* | mechanicas *L* 17 aut *(1)* angelo *(2)* intelligentia *L* 20 f. ratione; *(1)* inventa *(2)* quemadmodum trochoeides *(a)* parabolica cubica *(b)* parabolicae cubicae; *(3)* quemadmodum *L*

1 Vieta: *Supplementum geometriae*, 1593 (*VO* S. 240–257). — Fr. v. Schooten erwähnt die Ansichten von Viète in den *Commentarii in R. Descartes Geometriam*, *DGS* I, S. 169. 8 Cartesius: *Geometria*, 1659, *DGS* I S. 38 f. [Marg.] u. 50 (vgl. *DO* VI S. 412 u. 424). 11 f. Hugenio: *Horologium oscillatorium*, 1673; Romero: Leibniz wußte um O. Rømers Forschungen u. a. zu epizykloidisch geformten Rädern; vgl. *LSB* III,1 S. LXVII.

admodum; trochoeides parabolae quadrato-cubicae cuius dimensionem invenit Heuratius, analytica non erat ante quam ille curvae illius dedisset dimensionem.

Lineae ergo omnes a nobis descriptae geometricae aut mechanicae sunt. Geometricae aut analyticae aut tetragonisticae; analyticae sunt, quae sub calculum cadunt, ita ut relatio ordinatarum ad abscissas aequatione exprimi possit. Tetragonisticae, quae ad calculum tum demum revocari poterunt, cum dimensiones aut quadraturae quorundam curvilineorum invenientur. Analyticae sunt simplex (recta), quadraticae[,] cubicae, si aequationes ipsarum aestimes; in quibus scilicet duae incognitae quantitates eaedem aut diversae, altera in alteram ducuntur; cubicae etc. Tetragonisticae cuius gradus sint incertum est; (si impossibilis esset quadratura, essent nullius), tales sunt: trochoeides; evolutae; tetragonisticae aut sunt aut non sunt in nostra potestate, sunt evolutae et trochoeides, quas describere possumus; non sunt spirales; mechanicae sunt quae per puncta descriptae intelliguntur; ubi ne quidem rationem continuam describendi i n t e l l i g i m u s ; ut in logarithmica.

Aliter dividi poterat, lineas esse aut geometricas, aut physicas et mechanicas; geometricae, quorum descriptio a nobis intelligi pariter et fieri potest; physicae, quas describi intelligimus; ut a natura aut mente; mechanicae, quarum descriptionem neque posse possumus neque habemus. Sufficit interea geometricas a me appellari, quas uno tractu describi intelligimus; mechanicas, quas per puncta; analyticas esse eas geometricas, quae sub calculum cadunt. Unde intelligi potest non esse notam lineam [mechanicam] sub calculum cadere, nam logarithmicae quodlibet punctum analytice determinari potest; et tamen geometrica non est; cum nunquam revera, nec nisi ad sensum describi possit.

De trochoeidibus notandum est quandocunque curva alicuius figurae dimensioni analyticae subiecta est et trochoeidem eius fieri analyticam. Et tamen fatebitur mihi aliquis

1 parabolae *(1)* cubicae *(2)* quadrato-cubicae *L* 8 scilicet *(1)* rectangulum ne *(2)* duae *L* 10 f. sunt: *(1)* cycloeides, *(2)* trochoeides; evolutae; *(a)* mechanicae rursus duorum sunt generum, *(b)* tetragonisticae *L* 13 ne *(1)* comminisci quidem rationem continuam describendi possumus, ut in logarithmica *(2)* quidem *L* 15 f. geometricae, *(1)* quae a nobis intelliguntur et describuntur; physicae quae intelliguntur non describuntur *(2)* quarum *L* 17 posse *erg. L* 19 esse *(1)* eam speciem geometricarum *(2)* speciem earum geometricarum *(3)* eas *L* 20 non geometricam *L ändert Hrsg.* 21 punctum *(1)* geometrice *(2)* analytice *L*

1 parabolae: Leibniz meint die in H. van Heuraet, *Epistola de transmutatione curvarum linearum in rectas*, 1659, *DGS* I S. 517–520, als erstes Beispiel für die Rektifikationsmethode angeführte, heute als semikubisch bezeichnete Parabel $ay^2 = x^3$. 24 aliquis: z. B. Leibniz selbst; s. N. 39 S. 557 Z. 16–20.

descriptionem per constructionem tetragonisticam in illis casibus plerumque praelatum iri etiam analyticis etsi haberentur certe, etiamsi haberemus quadraturam circuli et omnium eius partium et sectionem angulorum universalem, quo facto cycloeidis descriptionem analyticam haberemus; mallemus tamen profecto volatione ista tam facili quam aliis regularum forte compositioribus describere.

Nota trochoeides parabolica dat sectionem rationis, et cycloeides linea dat sectionem anguli in data ratione; quae sunt illa duo magna geometriae desiderata.

Notabile est, cycloeidis ordinatarum differentias geometrice describi posse, est enim figura angulorum, quam voco. Si Cartesiani obiciunt fila vel chordas non per minima flecti, certe nec ellipsis descriptio ope fili geometrica erit. Et vero nulla plane regula perfecte recta haberi potest; nec tamen ideo regulam et lineam rectam repudiamus.

Data curva invenire aliam cuius sit trochoeides est invenire curvam differentiis figurae propositae homogeneam, imo paulo amplius quam homogeneam nimirum aequalem. Itaque quandocunque curva dimensionis capax est, eius trochoeidem habemus. Itaque generaliter data curva non habemus statim eius trochoeidem. Etsi data evolutionali habeamus evolutam. Ita habemus figuram cuius trochoeides sit parabola, ea est scilicet quadratrix parabolae cubicae Heuratianae.

Videndum an paraboloeides aut hyperboloeides eae, in quibus exponentes non sunt numeri rationales intelligi possint geometricae, v.g. $y^{\sqrt{2}} \sqcap x$. Non certe nisi ope trochoeidis meae.

Hyperbola est figura rationum; et $\dfrac{a}{\sqrt{2ax-x^2}}$ figura angulorum; et $\dfrac{ax}{\sqrt{2ax-x^2}}$ figura segmentorum.

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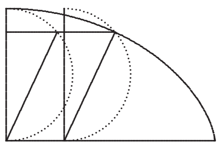
22 *Nach* segmentorum, *ohne Bezug zum weiteren Text:*

Cycloeides est 

1 constructionem *(1)* analyticam *(2)* tetragonisticam *L* 11 f. repudiamus. *(1)* Recte ipse Cartesius; *(2)* Data *L*

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10 ellipsis descriptio ope fili: R. DESCARTES, *Dioptrique*, 1637, S. 90 bzw. *Dioptrice*, 1644, S. 150 (vgl. *DO* VI S. 166 bzw. S. 623). 17 parabolae cubicae Heuratianae: s. o. Erl. zu S. 487 Z. 1.

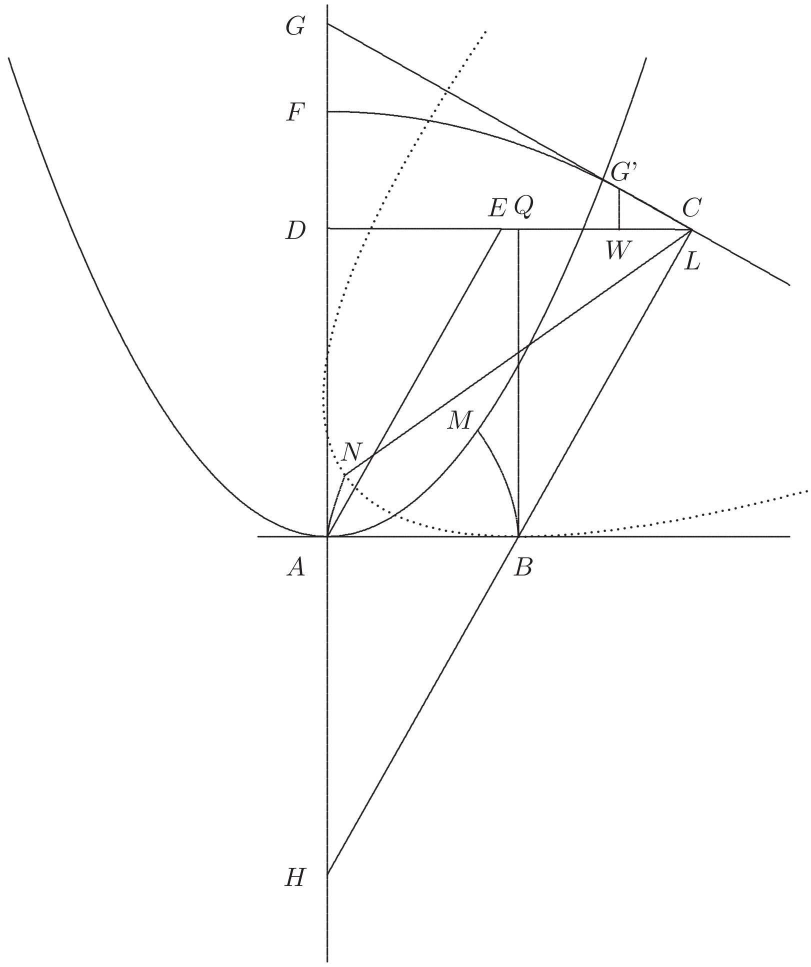


fig. 1.

Esto parabola tangens planum vertice A, eius axis $ADFG$. Sumto in axe puncto F. (forte utile erit AF. aequari lateri recto) describatur trochoeides; ex cuius puncto C.

1 fig. 1.: Leibniz verwendet in der Vorlage den Buchstaben G zur Bezeichnung von zwei verschiedenen Punkten. Zur besseren Unterscheidung wird der zweite in der Figur und im Text mit G' benannt.

ordinata in axem immobilem consideratum, parabolae in prima statione existentis AF, demittatur; iungatur et [BC.] punctum quo parabola nunc tangit planum, iuncta BC. erit perpendicularis ad curvam. $\dfrac{G'W}{WL} \sqcap \dfrac{a}{\sqrt{a^2+x^2}} \sqcap \dfrac{GD}{DC} \sqcap \dfrac{DC}{DH} \sqcap \dfrac{ED \sqcap CQ}{AD}$. Sumta ergo $FA \sqcap b$, et $FD \sqcap x$. erit $AD \sqcap b-x$. et fiet $ED \sqcap \dfrac{ba-ax}{\sqrt{a^2+x^2}}$. EC. autem aequalem curvae parabolae BM, appellare poterimus ψ fiet $CD \sqcap \dfrac{ba-ax}{\sqrt{a^2+x^2}} + \psi$. Sed inesse video errorem, [ratio $G'W$.] ad WL. non habetur. Omnia paulo consideratius agenda.

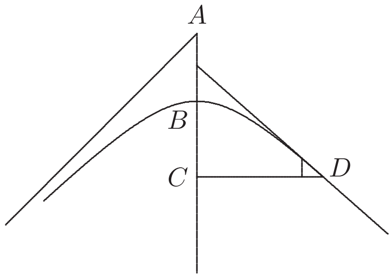


fig. 2.

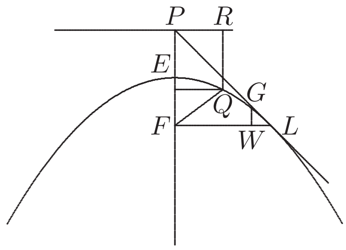


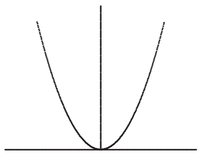
fig. 3.

$AC \sqcap y$. $CD \sqcap x$. A. centrum hyperbolae BD, cuius vertex B. erit $y \sqcap \sqrt{a^2+x^2}$. Esto alia curva EL, cuius axis EF. cuius tangens GL. infinite parva, recta autem GL. ducta in constantem quandam a, aequetur ipsi [AC.] ductae in GW. Patet rectangulum sub curva EL, et recta a. comprehensum aequari spatio BCD. GW, infinite parvam

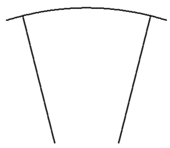
2 BD. *L ändert Hrsg.* 3 $\dfrac{\mathrm{G'W}}{\mathrm{WL}} \sqcap$ *(1)* $\dfrac{\mathrm{a}}{\mathrm{a}^2+\mathrm{y}^2}$ *(2)* $\dfrac{\mathrm{a}}{\sqrt{\mathrm{a}^2+\mathrm{x}^2}}$ *L* 6 rationem G. *L ändert Hrsg.* 8 f. y $\sqcap \sqrt{\mathrm{a}^2+\mathrm{x}^2}$. *(1)* GW $\sqcap \dfrac{\mathrm{a}^2}{\mathrm{a}^2}$ *(2)* Esto *L* 10 CD. *L ändert Hrsg.* 11 GW, *(1)* licet *(2)* infinite *L*

3 $\dfrac{G'W}{WL} \sqcap \dfrac{a}{\sqrt{a^2+x^2}}$: Die Gleichsetzung ist nicht zulässig. Leibniz erkennt den Fehler und setzt neu an.

appellemus 1. Ponamus $GL \frown a \sqcap \sqrt{a^2+x^2}$. ergo $GL \sqcap \dfrac{\sqrt{a^2+x^2}}{a}$. ergo $GL^2 \sqcap \dfrac{a^2+x^2}{a^2}$, unde si auferatur $GW^2 \sqcap 1$. restabit $\dfrac{x^2}{a^2}$, et erit $WL \sqcap \dfrac{x}{a}$. Summa autem omnium WL. aequatur FL. at summa omnium $\dfrac{x}{a}$[,] $\sqcap \dfrac{[x^2]}{2a}$. Ergo $FL \sqcap \dfrac{[x^2]}{2a}$. Ergo EL. curva est parabolica, cuius latus rectum $2a$. aequatur lateri recto hyperbolae transversum et rectum aequalia habentis.



[*Fig. 4*]



[*Fig. 5*]

Porro quoniam iam quaestio est, an punctum quod trochoeidem parabolicam describere debet in curva an in axe sumendum sit, video naturalissimum videri, ut sumatur in axe, neque enim video cur in curva aliud pro alio sumatur; praeterea, non inciperet ita trochoeides ab ipso axe, si punctum sumeretur in curva; itaque naturalissimum arbitror in ipso axe sumere parabolae focum. Ita enim mirabilis naturae orietur linea continuabilis in infinitum, sed involuta, et serpentina, ubi scilicet punctum parabolae foco oppositum praeterierit. Antea perpendicularis ad curvam ex puncto parabolae semper ducetur versus sinistrum, quando punctum foco oppositum in parabola, planum attigerit, tunc neque dextra erit neque sinistra sed perpendicularis; postea tendet in [dextrum]; unde habebit ibi flexum contrarium curva, si non involuta est.

In producto axe parabolae, fig. 3. sumatur $EP \sqcap$ quartae parti lateris eius recti $\sqcap EF$. erit $FP \sqcap a$. Et sumto in curva parabolae puncto quolibet Q. fiet $QR \sqcap QF \sqcap \dfrac{1}{2}a+y$

3 y^2 *L ändert Hrsg. zweimal* 4 aequatur *(1)* lateri tr *(2)* diametro *(3)* lateri *L* 7 iam *(1)* AN *(a)* $\sqcap$ *(b)* in fig. 1 $\sqcap \surd$ *(2)* quaestio *L* 13 Antea *(1)* curvae punctum semper erit a latere *(2)* perpendicularis ad curvam | ex puncto parabolae *erg.* | semper *L* 15 sinistrum *L ändert Hrsg.* 18 et *(1)* sumta in curva parabolae recta *(2)* sumto *L*

11 focum: Die folgenden Aussagen über die Gestalt der Kurve, die sich an fig. 1 orientieren, gelten nicht für die Trochoide des Brennpunktes, die eine nach oben offene Kettenlinie darstellt.

quae in fig. 1. $\sqcap AE \sqcap BC$ posita $AD \sqcap y$. in dicta fig. 1. eritque $AE^2 \sqcap \frac{a^2}{4} + ay + y^2$, unde si auferatur $AD^2 \sqcap y^2$, restabit: $\sqrt{\frac{a^2}{4} + ay}$ quae est applicata parabolae dimidii lateris transversi, nam pro $\frac{a}{4} + y$. posito z. fiet $\sqrt{az}$. cui si addatur curva parabolica $\sqcap \psi \sqcap EC$, fiet DC ordinata $\sqcap \sqrt{az} + \psi$.

Quemadmodum series inaequabiles non ideo negamus esse arithmeticas, quia calculo explicari possunt, etsi non aequatione; ita figurae quoque inaequabiles, quales sunt cycloeides et mesolaba, cum descriptione accurata exhiberi possint, non est negandum geometricas esse, etsi sub aequationem non cadant.

Veteres constructionem aliam esse geometricam dicebant, aliam organicam; hanc Cartesius recte explosit cum constet regulam et circulum organa quoque esse, etsi minus composita. Quicquid ergo organicum est, id geometricum est, modo supposita organi exactitudine, etiam lineam quandam veram seu geometricam, apparenti oppositam sequi necesse sit.

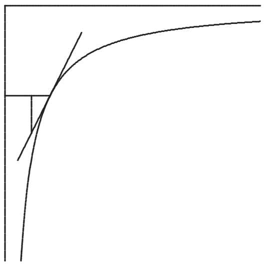
Si lineas eas appellamus geometricas, quarum ordinatae ad abscissas relationem habent calculo explicabilem; utique et logarithmica Renaldini ex eius et Gregorii et P. Pardies sententia erit geometrica, et curva quadratrix Wallisii: Sin ait Cartesius eas ideo non sufficere, ut calculo explicentur, sed ut aequatione, quod fieri non potest: Afferat distinctionis causam. Nam et series arithmeticae ut dixi non semper aequatione explicari possunt, v. g. progressio geometrica. Sin rationem affert; quod hae lineae quae sub calculum cadunt describi tamen nisi aequationis sint capaces nequeant: videt ergo non

2 applicata *(1)* eiusdem parabolae, ad aliam licet diametrum *(2)* parabolae *L* 5 Quemadmodum *(1)* figuras *(2)* series *L* 5 esse *(1)* geometricas *(2)* arithmeticas, *(a)* ita *(b)* quia *L* 13 f. sit. *(1)* Si C *(2)* Si *(a)* cum Cartesio *(b)* lineas *L* 20 capaces *(1)* ope calculi, *(2)* nequeant *L*

10 Cartesius: *Geometria*, 1659, *DGS* I S. 17–19 (vgl. *DO* VI S. 388–390). 15 f. logarithmica Renaldini: J. Gregory erwähnt in seiner Verteidigung der Logarithmuskurve, *Geometriae pars universalis*, 1668, Prooemium, die Mediceischen Kurven von Renaldini, bei denen es sich jedoch um spezielle Kurven zur Gleichungslösung handelt; Leibniz bezieht sich vermutlich auf die mißverständliche Darstellung des Sachverhalts durch die Rezension von Gregorys Werk in den *Philosophical Transactions* III Nr. 35 vom 18./28. Mai 1668 S. 685–688, insbesondere S. 685 f. — P. Pardies: *Élémens de géométrie*, 1671, préface. — curva quadratrix Wallisii: *Arithmetica infinitorum*, 1656, Scholium nach prop. 194 S. 196 (*WO* I S. 477).

calculum, sed describendi potestatem lineas reddere geometricas; et quasdam analyticas describi non posse, ac proinde non esse geometricas, vicissim quasdam geometricas calculari non posse atque adeo non esse analyticas.

$2ax \sqcap y^2$. $\not{2}al \sqcap \not{2}y^2$. Ergo $l \sqcap x$. Iam $\frac{l}{y} \sqcap \frac{\beta}{w}$. Ergo $w \sqcap \frac{y\beta}{x} \sqcap \frac{\beta\sqrt{2ax}}{x}$. eius quadrato addatur 1. fiet: $\frac{\sqrt{2ax+x^2}}{x}$. quae curva proinde pendet ex quad. hyp. Unde $2a\not{x} + x^{\not{2}} \sqcap \frac{x^{\not{2}}y^2}{a}$ sive: $\frac{2a^3}{y^2-a^2}$. Porro et $2ax \sqcap 2yl$. et $\frac{ax}{y} \sqcap l$. et pro x. ponendo $\frac{y^2}{2a}$. fiet: $\frac{ay^2}{2ay} \sqcap \frac{y}{2}$. Iam $\frac{l}{x} \sqcap \frac{\beta}{w}$. erit $w \sqcap \frac{2x\beta}{y} \sqcap \frac{2y^2}{2ay} \sqcap \frac{y}{a}$. et cuius quadrato addendo 1. fiet $\frac{y^2+a^2}{a^2}$. cuius radix $\frac{\sqrt{y^2+a^2}}{a}$. homogenea ordinatae hyperbolae et exprimens curvam parabolae. Interea valde notabile est, quod ita detexi eandem curvam diversis figuris homogeneam esse posse.

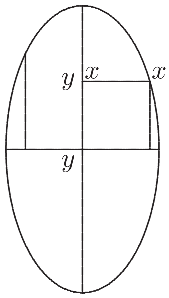


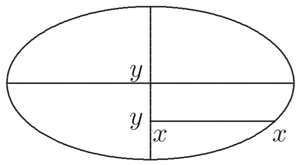
[*Fig. 6*]

Hyperbola: $a^2 \sqcap xy$. Unde $-\not{x}y \sqcap \not{x}l$. Iam $\frac{x}{l} \sqcap \frac{w}{\beta}$. Ergo $\frac{x\beta}{-y}$ seu $\frac{a^2\beta}{-y^2} \sqcap w$. cuius quadratum $\frac{a^4\beta^2}{y^4}$. Addatur β^2. fiet: $\sqrt{\frac{a^4\beta^2+\beta^2y^4}{y^4}}$. seu $\frac{\beta}{y^2}\sqrt{a^4+y^4}$. homogenea curvae hyperbolicae $\mp a^2 + x^2 \sqcap y^2$. Unde $\not{2}xl \sqcap \not{2}y^2$. Unde $l \sqcap \frac{y^2}{x} \sqcap \frac{\mp a^2+x^2}{x}$. Iam

1 describendi *(1)* possibilitat *(2)* potestatem *L* 7 $\sqcap \frac{y}{a}$. *(1)* Unde auferendo a^2 *(2)* et *L*

$w \sqcap \frac{y\beta}{l} \sqcap \frac{y\beta}{\frac{y^2}{x}} \sqcap \frac{x\beta}{y} \sqcap \frac{x\beta}{\sqrt{\mp a^2 + x^2}}$. Ad $\frac{x^2}{\mp a^2 + x^2}$ addatur 1. fiet: $\frac{x^2 \mp a^2 + x^2}{\mp a^2 + x^2}$, et $\sqrt{\frac{2x^2 \mp a^2}{\mp a^2 + x^2}} \sqcap \sqrt{1 \; \frac{[+x^2]}{\mp a^2 + x^2}}$. homogenea curvae hyperbolicae. Ope huius inventi semper plurium curvarum quadraturae alterius ad alteram reduci possunt.





[*Fig. 7*] [*Fig. 8*]

Ellipsis $a^2 - \frac{a}{b}y^2 \sqcap x^2$. Unde $-\frac{\not{2}a}{b}yl \sqcap \not{2}x^2$. sive $l \sqcap -\frac{bx^2}{ay} \sqcap \frac{-ba^2 + ay^2}{ay}$. Iam $w \sqcap \frac{y\beta}{l}$. Ergo $w \sqcap \frac{y^2\beta a}{-ba^2 + ay^2}$, sive cum est explicanda prius y. addatur 1. ad eius quad. fiet: $\sqrt{\frac{2y^4a^2 + b^2a^4 - 2ba^3y^2 \boxed{+a^2y^4}}{b^2a^4 - 2ba^3y^2b + a^2y^4}} \sqcap \frac{\sqrt{2y^4a^2 + b^2a^4 - 2ba^3y^2}}{\mp ba^2 \pm ay^2}$, sive $\frac{\sqrt{2y^4 + b^2a^2 - 2bay^2}}{y + \sqrt{ba} \frown y - \sqrt{ba}}$. Hinc puto inveniri posse tandem dimensionem curvae ellipticae. Additis iis quae de momento curvae ellipticae dicta sunt.

Curvae hyperbolicae valor pariter ex ellipticae quaeratur et explicata recta x. item sumta hac aequatione: $2av \mp \frac{a}{b}v^2 \sqcap z^2$. Quia curva hyperbolica est $\sqrt{\frac{2x^2 \mp a^2}{\mp a^2 + x^2}}$. ducatur in $\sqrt{\mp a^2 + x^2}$, eius momentum ex axe erit: $\sqrt{2x^2 \mp a^2}$. quod pendet ex quad. hyp. Iam

2 $\mp a^2$ *L ändert Hrsg.* 6 cum ... prius y. *erg. L* 9 f. sunt. *(1)* Momentum curvae hyperbolicae quaeratur *(2)* Curvae *L*

eadem curva et sic exprimitur: $\dfrac{\sqrt{a^4+y^4}}{y^2}$. Eius momentum ex asymptoto est: $\dfrac{\sqrt{a^4+y^4}}{y}$. Cuius momentum ex y. est $\sqrt{a^4+y^4}$. Momentum ex axe est $\dfrac{a^4}{y^2}+y^2$. Quod habetur pure. Iam $\dfrac{a^4+y^4}{y^2} \sqcap x^2$. Unde $y^4-x^2y^2+x^4 \sqcap x^4-a^4$. Unde $y^2 \sqcap \mp\sqrt{x^4-a^4}+x^2$. Quod momentum pendet ex quad. hyp. quia $\sqrt{x^4+a^4}$ pendet ex quad. hyp. ut ostendi plagula 9. pag. 2. huius schediasmatis. Iam puto has duas figuras $\sqrt{x^4-a^4}$, et $\sqrt{x^4+a^4}$ non differre, sed esse eandem, ut $\sqrt{x^2+a^2}$, et $\sqrt{x^2-a^2}$ non differunt. Cum ergo hoc modo tria habeamus momenta, figurae huius quae curvae hyperbolicae homogenea est, sequitur credo haberi ipsam figuram, et proinde et curvam, ex quad. hyp. Ipsae autem figurae ordinata y. erit $\sqcap \sqrt{\mp\sqrt{x^4-a^4}+x^2}$. In spem ex his venio dandae dimensionis curvae ellipticae et hyperbolicae. Memini Iac. Gregorium asserere, se dare centrum gravitatis curvae parabolae ex quad. circuli et hyperbolae; idem dicit Wallisius. Ego vero id dare possum ex sola quad. hyp. Hinc si non erramus aliquis dabit quad. circ. ex data quad. hyp. In eandem pene spem ex curvae ellipticae dimensione et momento erigor.

6 differunt. *(1)* Et y $\sqcap$ *(2)* Porro y $\sqcap$ $\sqrt{\mp\sqrt{\mathrm{x}^4-\mathrm{a}^4}+\mathrm{x}^2}$ quae *(3)* Cum *L* 7 habeamus *(1)* curvae *(2)* momenta *L* 9 $\sqrt{\mp\sqrt{\mathrm{x}^4-\mathrm{a}^4}+\mathrm{x}^2}$. *(1)* Iam quaeramus eius differentiam a $\sqrt{+\sqrt{\mathrm{x}^4-\mathrm{a}^4}-\mathrm{x}^2}$ *(2)* In *L*

---

3 $y^4-x^2y^2+x^4$: Leibniz erweitert bei der quadratischen Ergänzung mit x^4 statt mit $\dfrac{x^4}{4}$. Der Fehler wirkt sich auf die folgenden Gleichungen aus, jedoch nicht auf die grundsätzliche Überlegung. 4 ostendi: s. o. N. 38₁₁ S. 478 Z. 6 f. 10 Iac. Gregorium: *Vera circuli et hyperbolae quadratura*, 1668, Postscriptum S. 60. 11 Wallisius: Die für die Berechnung des Kurvenschwerpunkts erforderlichen Werte der Bogenlänge der Parabel und der Oberfläche ihres Drehkörpers bestimmt Wallis erstmals in *Tractatus duo*, 1659, S. 97–99 (*WO* I S. 554–556). Leibniz bezieht sich vermutlich — wie in *De hyperbola* (Cc 2, Nr. 612) — auf *Mechanica*, 1670–71, pars 2 cap. 5 prop. 31 S. [544]–555 und das Scholium S. 747 bis 753 (*WO* I S. 916–930).

Curva $\frac{a^2}{a^2+y^2} \sqcap x^2$. pendet ex quad. circuli. Est enim momentum anonymae lineae ex basi. Ergo et $\frac{y^2}{a^2+y^2} \sqcap x^2$. Earum summa: $\frac{a+y}{\sqrt{a^2+y^2}} \sqcap z$. et quadrando: $\frac{a^2+y^2+2ay}{a^2+y^2}$ [*bricht ab*]

2 *Über* Ergo et $\frac{y^2}{a^2+y^2}$: ϑ

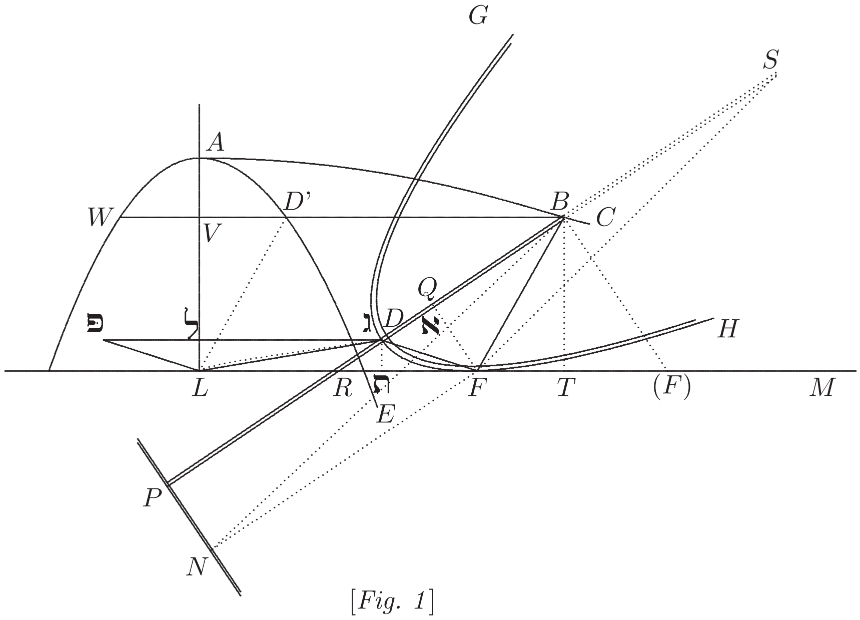
3 *Am Rande, gestrichen:* $\sqrt{\frac{a^2\beta^2}{y^2}-\beta^2} \sqcap \frac{\sqrt{a^2\beta^2-\beta^2y^2}}{y}$. Quadratum ergo est momentum hyperbolae ex diametro coniugata, et habebitur curva logarithmorum. $a^2-y^2 \sqcap x^2$. Ergo $y^2 \sqcap x^2+a^2$ [*!*]. Quadrata ergo omnium invenienda sunt. Ea vero habentur; habetur ergo curva quae sit homogenea elementis hyperbolae. Atque ita perfectam habebimus logarithmorum descriptionem.

8 ergo *(1)* momentum hyperbolae *(2)* curva *L*

38_{13}. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS UNDECIMA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 24–25. 1 Bog. 2°. 4 S. Überschrift ergänzt. Cc 2, Nr. 775 A tlw.

Plagula XI. schediasmatis de seriebus et summis.



[*Fig. 1*]

Veniunt in mentem res tanti momenti, circa curvam constructricem generalem, sive trochoeidem parabolicam, ut non possim non persequi.

7 parabolicam, | ut *streicht Hrsg.* | ut *L*

5 *Fig. 1*: Die flüchtig skizzierte Figur der Handschrift bildet die tatsächlichen Lage- und Größenverhältnisse nur stark verzerrt ab. Da Leibniz dies im Text kommentiert und punktuell korrigiert, wird die Figur möglichst vorlagengetreu wiedergegeben. — Zwei verschiedene Punkte, die Leibniz beide mit dem Buchstaben *D* bezeichnet, werden *D* u. *D*' benannt.

Descripta intelligatur trochoeides nostra, $[ABC]$. Parabola genitrix in situ descriptionis, $GDFH$. tangens planum LM. in puncto F. eius vertex D. umbilicus curvam ABC. describens $[B]$. FB. perpendicularis ad curvam. $D'L$. ipsi aequalis et parallela quia $D'B$. aequalis et parallela ipsi LF. Et FN. aequalis utrique ex proprietate parabolae, posito BP. esse eius latus rectum dimidium.

Et quia LM. tangit parabolam, ideo erit $DQ \sqcap DR$. seu QR. dupla ipsius QD. Axis DB. producatur in S. dum QS. fiet aequa ipsi BP. et ex nota parabolae proprietate SF. erit ipsi perpendicularis in F, et proinde angulus SFM. rectus et anguli SFL. et SFM. aequales, quamquam figura id non ita ferat, sed provideri talia ab initio tumultuaria descriptione non potuere. Ergo recta SF. parallela rectae AL. Iungatur BN. erit $BN \sqcap FS$. et triangula BPN. et SQF. aequalia et similia.

Ergo triangula SQF, (vel BPN.) et FQR. et BTR. erunt similia. Sunt autem BT. et SF. parallelae. Ex tot proprietatibus innumera poterunt duci theoremata memorabilia, sed et quadraturae (applicando ordinatas nunc ad axem parabolae, nunc ad axem trochoeidis,) et solidorum ac superficierum dimensiones. Sed quoniam nunc non nisi curvae aequationem quaerimus; illa in aliud tempus differemus.

Iam datur BR, posita enim $DQ \sqcap x \sqcap DR$, erit $BR \sqcap x + \frac{a}{2}$. ponendo latus rectum parabolae, $2a$.

Hinc iam facile habetur $FT \sqcap VD'$. Nam habetur $BF \sqcap QP \sqcap BR \sqcap \frac{a}{2} + x$.

---

11 *Am Rande:* Etiam FB. vel FN. vel QP. $\sqcap BR$

1 nostra, | ACD *ändert Hrsg.* |. *(1)* Descripta sit et parabola, genitricis dimidium habens latus rectum, AD'E. *(2)* Parabola *L* 2 GDFH. *(1)* eius axis *(2)* tangens *L* 2 f. umbilicus *(1)* B. describ *(2)* curvam ABC. describens | C *ändert Hrsg.* |. FB. perpendicularis *L* 6 QD. *(1)* Cumque rectae sint parallelae *(2)* Axis *L* 10 potuere. *(1)* Perpetuo *(2)* Ergo *(a)* anguli *(b)* triangula *(3)* Ergo *L* 10 f. BN $\sqcap$ FS. *(1)* et ei parallela *(2)* et *L* 15 f. quoniam *(1)* satis *(2)* nunc *(a)* nis *(b)* non nisi curvae *(a)* descriptionem quaerimus *(b)* aequationem quaerimus; *(aa)* ista *(bb)* missa *(cc)* ita *(dd)* illa *L* 17 datur *(1)* BT ad TR *(2)* BR, *(a)* quae est *(b)* posita *L* 18 f. 2a. *(1)* Iam $\frac{\mathrm{BT}}{\mathrm{BR} \sqcap \mathrm{x} + \frac{\mathrm{a}}{2}} \sqcap \frac{\mathrm{FQ} \sqcap \sqrt{2\mathrm{ax}}}{\mathrm{QR} \sqcap 2\mathrm{x}}$. Erit BT $\sqcap \frac{\mathrm{x} + \frac{\mathrm{a}}{2} \frown \sqrt{2\mathrm{ax}}}{2\mathrm{x}}$. FT $\sqcap$ TV*(!)*. Nam habetur FB *(2)* Hinc *L*
19 FT $\sqcap$ VD'. *(1)* Nam habetur VD' $\sqcap$ *(2)* Nam *L*

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16 differemus: vgl. *De trochoeidibus et relationibus reductarum ad ordinatas* (Cc 2, Nr. 828), datiert 24. Dezember 1674.

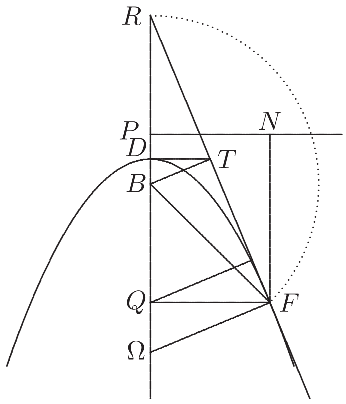


fig. 2.

$Q\Omega \sqcap 2DB \sqcap BP$.
Angulus $BFT \sqcap [NFT]$.

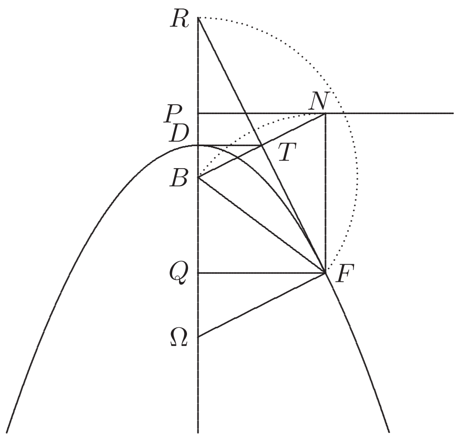
Iam sciendum est F potius cadere debere ibi ubi parenthesi (F) inclusum, et erit triangulum, RBF, isosceles, et $RT \sqcap TF$. Verus situs est in fig. 2. Tunc enim $BR \sqcap BF \sqcap FN \sqcap PQ \sqcap \frac{a}{2} + x$. Ergo DT parallela QF. Porro ut RF ad $[QR]$, id est, ut $\sqrt{4x^2 + 2ax}$ ad $2x$. ita $BF \sqcap BR \sqcap [RN] \sqcap PQ \sqcap D'L$ ad $RT \sqcap TF \sqcap D'V$ seu $\frac{a}{2} + x$. ad quartam. Ergo $\frac{\sqrt{4x^2 + 2ax}}{2x} \sqcap \frac{\frac{a}{2} + x}{D'V}$, et $D'V \sqcap \frac{ax + 2x^2}{\sqrt{4x^2 + 2ax}}$. Unde $\frac{a^2x^{\not 2} + 4ax^{\not 3 2} + 4x^{\not 4 3}}{4x^{\not 2} + 2a\not x} \sqcap D'V^2$.

498,19+499,4 $\sqcap \frac{a}{2} + x$ *(1)*, eius quadratum est $x^2 + ax + \frac{a^2}{4}$. Unde auferatur quadratum ipsius BT. *(a)* fiet: *(b)* nempe $\frac{x^2 + ax + \frac{a^2}{4}, \frown \not 2 a\not x}{2\not 4 x^{\not 2}}$ fiet: $\frac{2x^3 + 2ax^2 + \frac{a^2x}{2} - ax^2 - a^2x - \frac{a^3}{4}}{2x}$ sive $\frac{2x^3 + ax^2 + \frac{a^2x}{4} - \frac{a^3}{4}}{2x}$ $\sqcap$ quadrato ipsius FT. Sed suspicor calculi errorem, cum enim sit BF semper maior quam BT, | aut saltem aequalis *erg.* | hoc calculo posset fieri minor. *(2)* Nimirum *(3)* Iam *L* 2 NFB *L ändert Hrsg.* 5 fig. 2. *(1)* Unde et ratio patet erroris *(2)* Tunc *L* 6 ut *(1)* RQ $\sqcap$ 2x ad $\sqrt{2ax} \sqcap$ QF *(2)* RF ad | QF *ändert Hrsg.* |, id *L* 7 BN *L ändert Hrsg.* 8 D'V $\sqcap \frac{ax + 2x^2}{\sqrt{4x^2 + 2ax}}$. *(1)* Quae *(2)* Unde $\frac{a\not x + 2x^{\not 2}}{4x^{\not 2} + 2a\not x} \sqcap y^2$. *(3)* Unde $ax^2 + 2x \sqcap 4xy^2 + 2ay^2$, et $x \sqcap 2ay^2$ *(4)* Unde *L* et quia dividi potest, per $2x + a$, fiet: $x \frown a + 2x \sqcap 2D\text{'}V^2$, quae est aequatio ad hyperbolam. Unde patet RF esse velut diagonalem semiquadrati RBF. Iam si $\dfrac{a^2}{4} + ax + x^2$, auferatur: $\dfrac{xa}{2} + x^2$, restabit: $\dfrac{a^2}{4} + \dfrac{ax}{2}$, sive $\dfrac{a^2 + 2ax}{4}$ et $\dfrac{\sqrt{a^2 + 2ax}}{2} \sqcap LV \sqcap BT$. Iam LV, vocando y. fiet $4y^2 \sqcap a^2 + 2ax$, et erit $\dfrac{4y^2 - a^2}{2a} \sqcap x$. quem valorem ipsius x. inserendo in $D\text{'}V^2 \sqcap \dfrac{xa + 2x^2}{2}$, fiet: $D\text{'}V^2 \sqcap \dfrac{\dfrac{4y^2 - a^2}{2\not a} \frown \not a + \dfrac{16y^4 - 8a^2y^2 + a^4}{2\not 4a^2} \frown \not 2}{2}$ sive $[4a^2]\ D\text{'}V^2 \sqcap 4y^2a^2 \boxed{-a^4} + 16y^4 - 8a^2y^2 \boxed{+a^4}$. et fiet: $D\text{'}V \sqcap \dfrac{\not 2y}{\not 2a}\sqrt{[4]y^2 - a^2}$.

Habemus ergo naturam curvae nostrae, quantum licet aequatione expressam, nam abscissa LV, posita y, erit ordinata $[VB] \sqcap \dfrac{y\sqrt{[4]y^2 - a^2}}{a} \mp$ (puto $+$, sed examinandum in vero situ) $D\text{'}B$. $D\text{'}B$ autem iam aequetur curvae parabolicae. DF explicari analytice non potest, nisi per seriem numerorum.

1 potest, | fiet *streicht Hrsg.* | per *L* 2 patet *(1)* quadratum ipsius R *(2)* RF esse velut diagonalem *(a)*, et D' *(b)* quadrati *(c)* semiquadrati *L* 6 $4a^2$ *erg. Hrsg.* 6 f. D'V $\sqcap \dfrac{\not 2y}{\not 2a}$ | $\sqrt{2y^2 - a^2}$ *ändert Hrsg.* | *(1)* quae facile in aliam transmutan *(2)* sive D'V $\sqcap \dfrac{y}{a}\sqrt{y\sqrt{2} + a, \frown y\sqrt{2} - a}$ et $y\sqrt{2} - a$ vocando *(3)*. Habemus *L* 8 D'C *L ändert Hrsg.* 8 2 *L ändert Hrsg.*

2 semiquadrati RBF: RBF ist nur für den Spezialfall $x = \dfrac{a}{2}$ ein halbes Quadrat. 8 ordinata: Für die Trochoide des Brennpunkts gilt $VB = -D\text{'}V + D\text{'}B$.



[*Fig. 3*]

Inspice figuram hic ascriptam, in ea: $Q\Omega \sqcap BP \sqcap a$. $DP \sqcap DB \sqcap \frac{a}{2}$. $DR \sqcap DQ \sqcap x$. $2DT \sqcap QF \sqcap \sqrt{2ax}$. $FB \sqcap FN$. Ergo $BT \sqcap TN$. et recta [RTF], angulum BFN bissecat. Iam $FN \sqcap x + \frac{a}{2} \sqcap FB$. et $BR \sqcap DR$ seu $x, +DB$ seu $\frac{a}{2}$. ergo $FB \sqcap FN \sqcap BR \sqcap x + \frac{a}{2}$. Porro ad $QF^2 \sqcap 2ax$, addatur $Q\Omega^2 \sqcap a^2$, fiet: $\sqrt{a^2 + 2ax}$ [$\sqcap \Omega F$]. Et quia QD dimidia ipsius QR. seu DT. ipsius QF. erit TF dimidia ipsius RF,

500,10–501,2 numerorum. *(1)* Tantum addam AL aequari BD($\sqcap \frac{\mathrm{a}}{2}$) $\sqcap$ D'L. Ergo circulum *(a)* per *(b)* centro L; radio LD' descriptum transire per (A) locum ipsius A verum. Ergo iam invenio: *(2)* Unum iam superest monendum nempe BT *(a)* aequari *(b)* sive LV, aequari BQ et proinde quod optabam DQ abscissam parabolae, aequari AV, abscissae trochoeidis respondenti, ac proinde, *(aa)* descripta *(bb)* semiparabola DFH, translata in AW, nempe D in A, et F in W erectaque WVD'B, *(aaa)* curvam *(bbb)* rectam D'B fore curvae AW aequalem. *(aaaa)* Rectius *(bbbb)* Utilius ergo pro VL, incognitam *(aaaaa)* sumemus *(bbbbb)* abscissam sumemus AV, aequalem abscissae parabolicae x. Unum optarem, ut et VD' fieret $\sqcap$ VW. Sed hoc calculus negat, experiar denuo; a quadrato D'L $\sqcap \frac{\mathrm{a}^2 + 4\mathrm{ax} + \mathrm{x}^2}{4}$, | auferatur *streicht Hrsg.* | auferatur quadratum VL seu $\frac{\mathrm{a}^2 - 4\mathrm{ax} + \mathrm{x}^2}{4}$, restabit $\frac{8\mathrm{ax}}{4}$, seu 2ax. Ergo D'V $\sqcap\sqrt{2\mathrm{ax}}$, $\sqcap$ VW. quod est verissimum. Est ergo AD', vel AW, ipsa parabola generatrix. Huc usque ergo plane satisfactum est voto. — Sed iam incipio vereri, ne error sit in illa suppositione, quod scilicet DQ $\sqcap$ AV, DF $\sqcap$ VD', et BQ $\sqcap$ VL $\sqcap$ BT. *(3)* Inspice *L*   3 BTN *L ändert Hrsg.*   5 f. $\sqrt{\mathrm{a}^2 + 2\mathrm{ax}}$ | $\sqcap \Omega$F *erg. Hrsg.* | *(1)*, eiusque dimidium *(2)*. Et *L* et BT ipsius ΩF. Ergo $BT \sqcap \dfrac{\sqrt{a^2+2ax}}{2}$ quae in figura prima primae paginae $\sqcap$ [LV]. Eius quadratum $\dfrac{a^2+2ax}{4}$ auferatur a quadrato ipsius BF, quod est $\dfrac{4x^2+4ax+a^2}{4}$, restabit: $\dfrac{4x^2+2ax}{4} \sqcap \dfrac{2x^2+ax}{2} \sqcap TF^2$ ut supra. Item a BF^2, seu $\dfrac{4x^2+4ax+a^2}{4}$ auferatur $QF^2 \sqcap \dfrac{8ax}{4}$, fiet: $\dfrac{4x^2+4ax-8ax+a^2}{4}$, $\sqcap BQ^2$, et erit $BQ \sqcap \dfrac{\mp 2x \pm a}{2}$ sive erit BQ. differentia inter x et $\dfrac{a}{2}$, quod et dudum constat[,] est enim utique BQ differentia inter DQ et DB. Unde satis apparet non esse BT aequale ipsi BQ nec ideo in figura primae paginae prima $AV \sqcap DQ$. Quare nec AD' curva est parabolica generatrici eadem; ut in cycloide evenit.

Nunc BT appellando y, fiet:

$\dfrac{a^2+2ax}{4} \sqcap y^2$. et $x \sqcap \dfrac{4y^2-a^2}{2a}$. et $x^2 \sqcap \dfrac{16y^4-8a^2y^2+a^4}{4a^2}$, inseratur valori ipsius TF^2, fiet: $\dfrac{16y^4-8a^2y^2\boxed{+a^4}}{[4]a^2} + \dfrac{4y^2\boxed{-a^2}}{[4]} \sqcap \dfrac{16y^4-4a^2y^2}{[4]a^2}$ ut supra.

1 f. paginae $\sqcap$ | AV. *ändert Hrsg.* | *(1)* Ab eius quadrato $\dfrac{\mathrm{a}^2+2\mathrm{ax}}{4}$ auferatur quadratum ipsius DB, quod est $\dfrac{\mathrm{a}^2}{4}$ *(2)* Eius *L*   8 f. evenit. *(1)* Si ab $\mathrm{x}+\dfrac{\mathrm{a}}{2}$ auferatur TF *(2)* Nunc *L*   11 fiet: | $\dfrac{16\mathrm{y}^4-8\mathrm{a}^2\mathrm{y}^2+\boxed{\mathrm{a}^4}}{2\mathrm{a}^2}$ *ändert Hrsg.* | *(1)* $+4\mathrm{y}^2\boxed{-\mathrm{a}^2} \sqcap \dfrac{16\mathrm{y}^4}{2\mathrm{a}^2}$ et erit TF $\sqcap \dfrac{4\mathrm{y}^2}{\mathrm{a}\sqrt{2}}$ sive $\sqcap \dfrac{2\mathrm{y}^2\sqrt{2}}{\mathrm{a}}$; unde aequatio: $\mathrm{y}^2 \sqcap \dfrac{\mathrm{aTF}}{2\sqrt{2}}$ *(a)* cuius latus *(b)* quae indicat curvam AD'E in fig. 1 pagina prima huius plagulae esse parabolam quidem, sed cuius latus *(aa)* transversum *(bb)* rectum $\dfrac{\mathrm{a}}{2\sqrt{2}}$. est ad 2a latus rectum parabolae generatricis, $\dfrac{\mathrm{a}}{4\sqrt{2}}$ ut 1 ad $4\sqrt{2}$, seu ut 1 ad Rq 32. Cumque omnis parabola alteri sit similis, modo et similiter posita sit, (quod secus est in ellipsi et hyperbola latus rectum transversumque aequalia habente) ac proinde omnis parabola possit ita collocari, ut alteri parallela reddatur, seu ut omnis recta uni perpendicularis producta etiam sit alteri perpendicularis; ideoque intelligi potest hanc novam parabolam eodem, cum genitrice trochoeidis, seu data parabola, tempore describi posse; quod utile est, ne diversae descriptiones turbent exactitudinem. *(2)* | $+\dfrac{4\mathrm{y}^2\boxed{-\mathrm{a}^2}}{2} \sqcap \dfrac{16\mathrm{y}^4-4\mathrm{a}^2\mathrm{y}^2}{2\mathrm{a}^2}$ *ändert Hrsg.* | ut *L*

3 ut supra: S. 500 Z. 1.   11 ut supra: S. 500 Z. 5.

Ergo ut concludamus ut ante: abscissa LV posita y, ordinata trochoeidis parabolicae seu $[VB]$ erit $\sqcap \frac{y}{a}\sqrt{[4y^2-a^2]}+D$א$F$ posito DאF esse curvam parabolicam et nota $D'V$ esse $\sqcap \sqrt{\frac{y,y,4y+2a,4y-2a}{[4]a^2}}$.

Hactenus de trochoeide nostra quae describitur ab umbilico. Superest, ut consideremus an commodior illa futura sit, quae describitur a parabolae vertice, qualis est LגD ad quam recta FD erit perpendicularis unde ordinata ducatur לג producenda dum ipsi Lפ parallelae ad FD, occurrat in פ erit Dפ arcui DאF aequalis. Ordinata ergo Dל erit curva minuta recta [פל] cuius superest ut quaeramus valorem.

Habetur valor ipsius FD. Nam QD posita x, fiet $FD \sqcap \sqrt{x^2+2ax}$. Porro ל$L \sqcap D$ת. facile habetur, nam ut $RF \sqcap \sqrt{4x^2+2ax}$, ad QF seu $\sqrt{2ax}$ ita $RD \sqcap x$, ad Dת. Ergo $\frac{D\text{ת}}{x} \sqcap \sqrt{\frac{2ax}{4x^2+2ax}}$ et fiet: Dת $\sqcap \sqrt{\frac{2ax^2}{4x+2a}} \sqcap x\sqrt{\frac{\not{2}a}{2\not{4}x+\not{2}a}} \sqcap y$. ponendo Dת $\sqcap y$. et fiet: $x^2a \sqcap 2y^2x+ay^2$. et fiet: $x^2-\frac{2y^2}{a}x+\frac{y^4}{a^2} \sqcap \frac{y^4}{a^2}+y^2$ et erit $\mp x \pm \frac{y^2}{a} \sqcap \frac{y}{a}\sqrt{y^2+a^2}$, sive $x \sqcap \mp\frac{y}{a}\sqrt{y^2+a^2}+\frac{y^2}{a}$.

1 ante *(1)* recta *(2)* abscissa *L* 2 y^2-2a^2 *L ändert Hrsg.* 2 D'C *L ändert Hrsg.* 3 2 *L ändert Hrsg.* 8 Lפ *L ändert Hrsg.* 9 posita x, | fiet: *streicht Hrsg.* | fiet *L* 10 f. Ergo $\frac{D\text{ת}}{x} \sqcap$ *(1)* $\sqrt{\frac{2\not{4}x^{\not{2}}+\not{2}a\not{x}}{\not{2}a\not{x}}}$ vel Dת $\sqcap x\sqrt{\frac{2x+a}{a}} \sqcap$ y ponendo | igitur *gestr.* | Dת $\sqcap$ y. Itaque $2x^3+ax^2 \sqcap ay^2$. Iam Lל$^2 \sqcap \frac{2x^3+ax^2}{a}$ subtrahatur a פל^2 seu a x^2+2ax, residui quadratum (videndum ne error calculi insit, nam videtur aliquando id quadratum minus nihilo fieri posse) *(2)* $\sqrt{\frac{2ax}{4x^2+2ax}}$ *L*

1 ut ante: vgl. Erl. zu S. 500 Z. 8.

Porro פל $\sqcap \dfrac{-2ax^2}{4x+2a} + x^2 + 2ax$ (seu $FD^2[-]D$ת^2) $\sqcap$ $\dfrac{-2ax^2 + 4x^3 \; [+\, 4a^2x + 2ax^2 + 8ax^2]}{4x+2a}$; in quo si substituatur valor ipsius x, habebitur itaque aequatio satis composita, explicans valorem ipsarum פל per ipsas Lל abscissas. Ut proinde satius tandem videatur trochoeide uti ab umbilico descripta. Poterit tamen examinis causa describi utraque, quoniam id fit eodem tempore.

Descriptio autem ope fili ex materia non facile distendibili, in asseris zonam parabolicam tangentis incisuram se indentis, ponderis appensi tendentisque vi; assere interea et tabulam fixam in quam stylus agit, circumferente; perfectissime describetur; cum eadem opera et evolutionalis parabolica in pariete fixo a stylo asseri infixo possit describi. Stylus autem acicula esse potest in materia molluscula lineam inscribens.

Sed satis de descriptione veniamus ad usum. Ubi unum ante adiciem lineam istam pantometram aut similem esse absolute necessariam ad perfectionem geometriae[,] sunt enim problemata analytice soluta, seu quae calculo fieri possunt, et quorum tamen constructio geometrica per lineas analyticas non datur; qualia sunt: rationem secare in ratione duorum numerorum irrationalium. Haec autem problemata impossibile est solvi geometrice nisi per lineas non analyticas. Fatenda est ergo necessitas linearum non analyticarum in geometria. Praeterea [impossibile] est lineam inveniri analyticam quae sit omnium graduum simul; qualis ista est: itaque tot opus est curvis Cartesiana construendi methodo, quot sunt genera aequationum, cum linea pantometra sola sufficiat omnibus.

Vid. part. XII.

1 f. (seu FD^2| + Dת^2) $\sqcap$ $\dfrac{-2ax + 4x^3 + 2\text{④}\, a^2x}{4x+2a}$ *ändert Hrsg.* |; *(1)* et fiet: פל $\sqcap$ *(2)* in *L*

17 Praeterea | impossibilis *ändert Hrsg.* | est lineam inveniri *(1)* geometricam *(2)* analyticam *L*

18 f. Cartesiana construendi methodo: *Geometria*, 1659, *DGS* I S. 84 f. u. 105 f. (vgl. *DO* VI S. 463 f. u. 485).

38$_{14}$. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS DUODECIMA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 26–27. 4 S. Fig. 1, 2, 4, 6 mit Vorstufen und Korrekturen.
Cc 2, Nr. 775 A tlw.

Pars XII schediasmatis de quadraturis figurarum et summis serierum.

Nunc ad usum figurae nostrae pantometrae veniamus. Vide hic fig. 1.

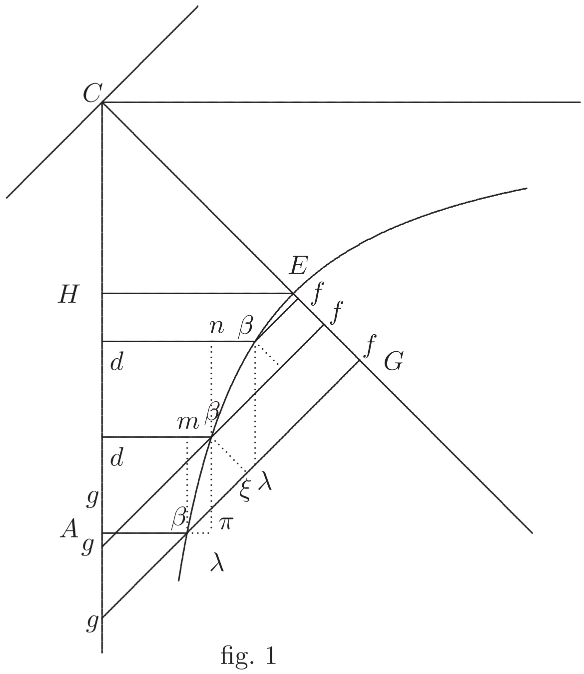


fig. 1

Si Ad sint inter se ut numeri, spatia $Ad\beta$. erunt inter se ut eorum logarithmi per inventa Gregorii a S. Vincentio. Sit centrum C. vertex E. axis Cf. Ducantur ordinatae βf. ad axem. Et eaedem producantur dum asymptotis occurrant in g.

9 axis *(1)* | EF. *streicht Hrsg.* | *(2)* Cf. *L*

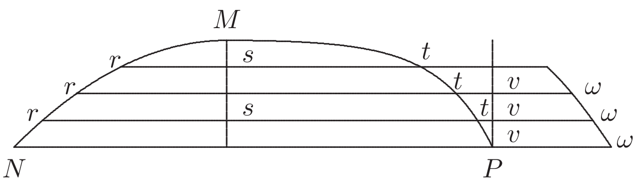
7 fig. 1: Leibniz hat eine erste Fassung der Figur gestrichen und in der zweiten Punktbezeichnungen geändert, den Text aber nicht konsequent angepaßt. Es wird an die Bezeichnungen der gültigen Figur angeglichen. 9 S. Vincentio: *Opus geometricum*, 1647, Buch VI prop. 125–130 S. 594–597.

Spatium $\beta Ad\beta$. est rectangulum $Ad\beta$. demto curvilineo $\beta\lambda\beta$. Esto dd. vel $dA \sqcap b$. $[AC] \sqcap z$. $CH \sqcap a$. erit $[dm] \sqcap \dfrac{a^2}{z}$. et $[Adm.]$ rectangulum erit: $\dfrac{ba^2}{z}$.

Investiganda quin quantitas ipsius f. Patet triangulum gfC. esse simile triangulo $gA\beta$. Ergo $\dfrac{gA}{A\beta \sqcap \dfrac{a^2}{z}} \sqcap \dfrac{gf}{fC}$. Imo $Ag \sqcap A\beta$. et $gf \sqcap Cf$. et $g\beta \sqcap \sqrt{\dfrac{2a^4}{z^2}} \sqcap \dfrac{a^2}{z}\sqrt{2}$. et $gf \sqcap \dfrac{a^2}{z}\sqrt{2} + \beta f$. At $\beta f \sqcap \sqrt{2a^2+x^2}$. ponendo $CE \sqcap a\sqrt{2}$. et $Cf \sqcap x$. ut constat.

Eritque: $\dfrac{a^2}{z}\sqrt{2} + \sqrt{2a^2+x^2} \sqcap a\sqrt{2}, + x$. Adeoque z. habetur absolute $\sqcap \dfrac{a^2[\sqrt{2}]}{a\sqrt{2}+x-\sqrt{2a^2+x^2}} \sqcap z$. Contra si x. investigare velis, retenta z, fiet: $\sqrt{2a^2+x^2} \sqcap a\sqrt{2}+x-\dfrac{a^2}{z}\sqrt{2}$. Unde $\boxed{2a^2}\boxed{+x^2} \sqcap \boxed{2a^2}+2ax\sqrt{2}\boxed{+x^2}, \boxed{-\dfrac{2a^2\sqrt{2}\sqrt{2}}{z}} - \dfrac{4a^2}{z} - \dfrac{2a^2x\sqrt{2}}{z} + \dfrac{2a^4}{z^2} \sqcap 0$. sive: $2axz^2\sqrt{2} - 4a^2z - 2a^2xz\sqrt{2} + a^4 \sqcap 0$. et $x \sqcap \dfrac{4a^2z-a^4}{2az^2\sqrt{2}-2a^2z\sqrt{2}}$. Iam pro z. pone $z-b$. fiet: $\dfrac{4a^2z-4a^2b-a^4}{2az^2-4azb\sqrt{2}+2ab^2-2a^2z\sqrt{2}+2a^2b\sqrt{2}}$. quarum duarum x. differentia utique est ff.

Iam spat. $\beta Ad\beta \sqcap \square A\lambda\beta -$ spat. $\beta\lambda\beta$. sed spatium $\beta\lambda\beta \sqcap$ spat. $\beta ff\beta - \square\, ff\xi - \triangle\, \beta\xi\pi + \triangle\, \pi\lambda\beta$. Ergo spat. $\beta Ad\beta \sqcap \square\, A\lambda\beta -$ spat. $\beta ff\beta + \square\, ff\xi + \triangle\, \beta\xi\pi - \triangle\, \pi\lambda\beta$.

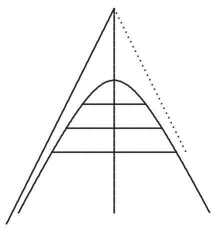


[*Fig. 2*]

2 AG *L ändert Hrsg.* 2 gm *L ändert Hrsg.* 2 Agm *L ändert Hrsg.* 7 $\sqrt{2}$ *erg. Hrsg.*

5 $\sqrt{2a^2+x^2}$: Richtig wäre $\sqrt{x^2-2a^2}$. Leibniz rechnet mit dem falschen Wert weiter; hinzu kommen Flüchtigkeitsfehler in Z. 8, Z. 9 u. Z. 10, welche die Rechnung bis Z. 10 beeinträchtigen.

Quod si iam inveniri potest linea curva cuius elementa rr procedunt ut ff. Et ordinatae trochoeidis (quae figura superflua diminuta sit) st, procedant ut summae elementorum, seu ut curvae portiones Mr tunc procedent etiam ut spatia $Eff\beta\beta E$. Sed hoc loco contrarium postulatur ut procedant ut spatia $Gff\beta\beta G$, sciendum ergo hoc modo procedere trochoeidis huius complementa tv. Atque ita id praeterea eveniet praeclarissimum, ut ordinatae tv sint purae et simplices aequales curvae, est enim $tv \sqcap Nr$ nulla extraneae figurae alioquin trochoeidem turbantis intermixtione. Iam ad tv adiungantur $v\omega$ quae scilicet procedant ut differentiae $Add\beta\beta A$ et $Gff\beta\beta G$ quod utique fieri potest, quoniam earum differentiarum progressio opinor erit constans et aequatione exprimi poterit; ideoque $t\omega$ procedent ut logarithmi numeris naturalibus prout procedentibus non quidem ut Pv, vel Gf attamen ut Ad.



[*Fig. 3*]

Ponamus iam istam ita descriptam haberi curvam cuius ordinatim applicatae, alterius curvae descriptae ordinatim applicatis auctae procedant ut logarithmi. Sane ita facile erit rationem secare in data ratione; sed quaeritur a me quiddam maius. Nempe constructio omnium aequationum etiam affectarum, quod videamus esto $\dfrac{b}{a+\dfrac{cx}{a}} \sqcap \dfrac{x^2}{d}$. Id est, quaerenda est linea ex asymptoto hyperbolae quo dixi modo abscissae [Ad] seu logarithmica $t\omega$ ut differentia inter logarithmum rectae cognitae b, et logarithmum rectae a, logarithmo c, et x summa auctum, et logarithmo ipsius a minutum, aequetur logarithmi dupli ipsius x, differentiae a logarithmo d, ipso minore quae ut pateant clarius aequatio analytica convertatur in logarithmicam, ponendo literas $b.\,a.\,c.\,x.\,d.$ significare iam non ipsas lineas seu numeros sed eorum logarithmos, et fiet: $b\boxed{-a}-c\boxed{+a}-x \sqcap 2x-d$, fietque $x \sqcap \dfrac{b-c+d}{[3]}$. Sed videtur hic subesse error; nam [logarithmi] ipsius

1 cuius *(1)* ordinatae procedunt ut spatia Efβ *(2)* elementa *L* 5 huius *(1)* elementa *(2)* complementa *L* 7 f. ad *(1)* vω *(2)* tv adiungantur vω *(a)* necessariae scilicet ut solae *(b)* vel etiam ab altero *(c)* necessariae scilicet *(d)* quae *L* 8 Gff$\beta\beta$G *(1)* et tot *(2)* quod *L* 11 vel Gf *erg. L* 13 applicatae *(1)* sunt homo *(2)* utut *(3)* alterius *L* 19 abscissae | AG *ändert Hrsg.* | seu ... tω *erg.* | ut *L* 19 inter *(1)* ordinatam *(2)* logarithmum *L* 19 b, | et *streicht Hrsg.* | et *L* 24 2 *L ändert Hrsg.* 24 error; *(1)* logarithmus enim non *(2)* nam *(a)* logarithmus inveniendus est *(b)* | logarithmus *ändert Hrsg.* | ipsius *L*

b et d habentur; sed logarithmus non sumendus est ipsius $a.\,c.\,x.\,a.$ separatim, sed logarithmus quaerendus est ipsius totius $a+\dfrac{cx}{a}$. Nimirum ex logarithmis partium non componitur logarithmus totorum. Nisi velis numeros illos invenire quorum logarithmi sint ipsi naturales; quorum numerorum potestates eo modo invenies. Sed hoc nihil ad rem pertineret.

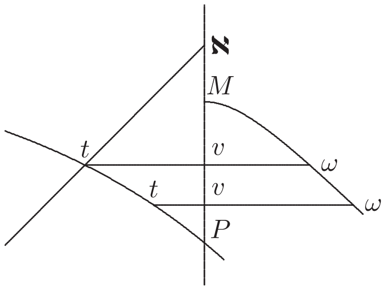
An ita? Pro x naturali cuius logarithmica quaeritur seu ex quaesitis Ad, et $t\omega$, rem eo reducamus, ut quaeratur fg et vt, eodem modo et b, et d, reducamus et $a+\dfrac{c}{a}x$, et x, reducendo numero naturales et eorum logarithmos: ad numeros artificiales et eorum quasi logarithmos; quod una semper methodo constante fit, quo facto aequatio habebitur correcta: $\boxed{b}-\boxed{a+\dfrac{c}{a}x}\ \sqcap\ \boxed{2x}-\boxed{d}$. Numeris inclusis, propositorum quasi logarithmos significantibus; additis scilicet vel subtractis antea illis superfluis nempe rectis $v\omega$, quae calculo semper haberi possunt vel sic[:] ipsius b, ipsam $v\omega$, appellemus (β), ipsius $a+\dfrac{c}{a}x$, appellemus $\alpha+\dfrac{\kappa}{\alpha}\xi$, et ipsius d, vocemus δ, et fiet aequatio correcta:

$$b-\beta-\underbrace{a+\frac{c}{a}x}+\boxed{a+\frac{c}{a}x}\ \sqcap\ 2x-2\xi-d+\delta.$$

Ubi patet incognitas esse quatuor: $\underbrace{a+\dfrac{c}{a}x}$, $\boxed{a+\dfrac{c}{a}x}$, x,ξ, logarithmos partim supplementa logarithmica. Et video si aequationem initio propositam in aliam mutare velis analogiam[,] multiplicari rursus incognitas. Non ergo sic exitus.

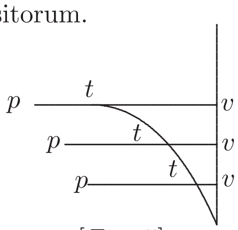
An ita[:] rectam ducamus $\aleph t$ quae secet curvam Pt in t, ipsa ωt, per lineam rectam ex sumta Ad incognita, habebit valorem quendam explicabilem, quia ipsa tv ordinata trianguli, et $v\omega$ ordinata curvae analyticae. Eadem recta iam habet et alium valorem, ut sit logarithmus ipsius Ad.

10 $-\boxed{\mathrm{d}}$. *(1)* eritque x $\sqcap$ *(2)* Numeris *L* 15 patet *(1)* tres fieri incognitas x, $\boxed{\mathrm{a}+\dfrac{\mathrm{c}}{\mathrm{a}}\mathrm{x}}$ *(2)* incognitas *L* 16 logarithmica. *(1)* Quare aequatio initio proposita mutetur in hanc: *(2)* Et *L* 18 f. rectam *(1)* ex sumta P *(2)* ex *L* 19 ipsa tv *(1)* recta cognita, et *(2)* ordinata *L*



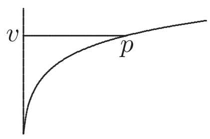
[*Fig. 4*]

Ecce ergo aequationem factitiam inter logarithmum, et alias quasdam cognitas, cuius ope videndum an possint solvi problemata; sed dubito, quia non eam intrant logarithmi quales $a + x$. terminorum compositorum.

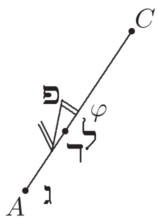


[*Fig. 5*]

Videndum ergo illud est an non fieri possit, ut linea *vp* fiat perfecte logarithmica, quod fieret retroagendo nonnihil ipsam parabolam genitricem inter volvendum id est re ad nostram descriptionem applicata, prorsum seu dextrorsum agendo separato motu, tabulam in qua logarithmica describi debet, retroagetur vero quantum postulat summa spatiorum complementalium, idque per applicationes regularum constantium, eo scilicet modo aliae quoque curvae describuntur. Qui motus ab ipso evolutionis motu pendebit.



[*Fig. 6*]



[*Fig. 7*]

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1 *Fig. 4*: Leibniz hat eine erste Fassung der Figur gestrichen und in der zweiten Punktbezeichnungen geändert, den Text aber nicht konsequent angepaßt. Es wird an die Bezeichnung der gültigen Figur angeglichen. 6 linea *vp*: Leibniz meint die Kurve mit der Ordinate *vp*.

Ponamus ergo haberi lineam vp vere logarithmicam; resumta aequatione superiore $b - \underbrace{a + \frac{c}{a}x} \sqcap 2x - d$. vel $d + b - \underbrace{a + \frac{c}{a}x} \sqcap [2]x$. In adiuncta hyperbolae asymptoto cuius initium arbitrarium A, centrum C sumatur דג $\sqcap a$. Praeterea ex ג ducatur recta פג angulum ad asymptoton faciens talem ut פד posita x, ipsa דφ sit $\frac{c}{a}x$. et ipsa גφ erit $a + \frac{c}{a}x$. eique respondens vp vel $t\omega$ erit logarithmus $a + \frac{c}{a}x$. Iam ipsa ד translata cogitetur in ipsam AC, et ד incipere ab A, et פ esse etiam in recta AC v. g. פד translatam in Aל.

Itaque si regula sit ex A, versus C tendens indefinita mobilis in fixo assere AC. et in φ fixus sit palus circa quem chorda ita ut dum regula alia פφ mobilis in ipsius פד assere descendit, regula Aל eodem motu etiam descendat, dum autem descendent aliarum regularum applicationibus signabuntur earum logarithmi aut potius a datis logarithmis differentiae; et quando eveniet, ut concurrant duae regulae in unum, ibi vero habebitur aequationis radix.

Sed quid si esset: $\dfrac{b}{e + xa + \frac{c}{a}x^2} \sqcap \dfrac{x^2}{d}$. Eo casu id profecto non ita succederet, quomodo enim logarithmum ipsius x^2, per regulam designabimus. Consideranda aequatio proposita: $x^4 + ax^3 + ex^2 - bd$, ut quantitas composita ex rectarum quarundam differentiis ab ipsa x vel eius summis: sed nec hoc procedit, quia ipsa x universalis est quantitas imaginaria.

An ita: quaerendae sunt quatuor lineae naturales quarum logarithmorum summa bd, ipsarum linearum summa a: summa numerorum naturalium logarithmo cuilibet logarithmis duorum quorumlibet combinatis aequali respondentium sit e. sed difficiliora sunt ista, quam ut spes sit vinci posse; nisi quadam arte artibus Vietae simili utamur; ut

2 2 *erg. Hrsg.* 2 f. cuius ... centrum C *erg. L* 10 logarithmi *(1)* intersectione quarundam regularum et rectae p *(2)* aut potius *(a)* cum datis *(b)* a *(c)* a datis *L* 14 designabimus *(1)*, nisi ita addendo *(2)*. *(a)* Ista ergo difficillima *(b)* Videndum, an *(c)* Consideranda *L* 18 sunt *(1)* tres lineae, *(2)* quatuor lineae *(a)* in logarithmis, *(b)* naturales *L* 19 f. summa a: *(1)* summa logarithmorum duorum quorumlibet *(2)* summa numerorum naturalium | logarithmo cuilibet *erg.* | logarithmis duorum quorumlibet *(a)* aequali *(b)* combinatis *L*

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21 Vietae: *De numerosa potestatum*, 1600 (*VO* S. 162–228).

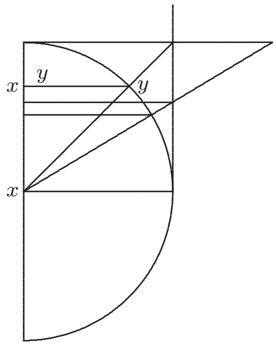
scilicet quia ut ille ab extractione incipit, caetera multiplicationibus subtractionibus et additionibus peragit; ita nos incipiendo a divisione logarithmorum, caetera additionibus etc. Sed video nec hoc procedere, nam in numeris fiet semper ob alias causas tractabilior.

Malum omne ex eo quod ut logarithmi repraesentant quidem extractionem radicum, divisione; multiplicationem additione et divisionem substractione ipsorum; sed nihil habent, quo repraesentent additionem et subtractionem naturalium. Quemadmodum naturales nihil habent quo repraesentent extractionem radicum ex logarithmis.

Sumantur numeri quorum logarithmi sint ipsi naturales; hi naturales naturalium, multiplicantur additione naturalium, et involvuntur additione logarithmorum.

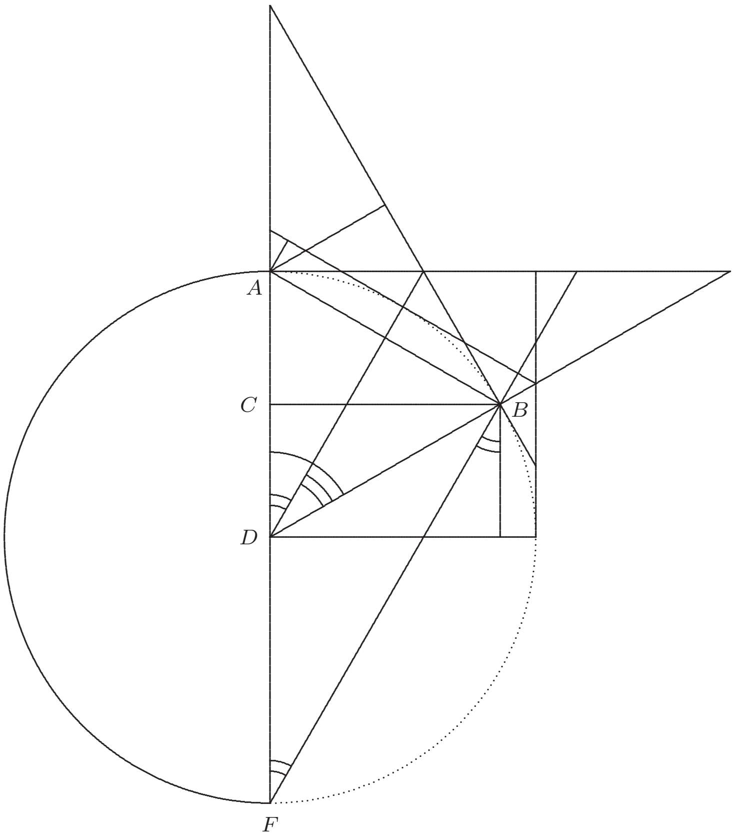
Itaque si detur $\dfrac{b}{a+\dfrac{c}{a}x} \sqcap \dfrac{x^2}{d}$, et horum loco aequatio similis fiat inter logarithmos; faciendo: $b - \underbrace{a+\dfrac{c}{a}x} \sqcap 2x - d$.

Si invenire posset curva ipsis elementis hyperbolae ad asymptoton homogenea haberetur descriptio curvae logarithmorum; nempe $\dfrac{a^2-x^2}{x^2}, \sqcap \dfrac{\sqrt{a^2-x^2}}{x}$ cuius quaeritur dimensio seu quadratura: aequatio est: $a^2 - [x^2] \sqcap x^2y^2$. quae pendet ex quadratura circuli.



[*Fig. 8*]

4 quidem *(1)* additionem *(2)* extra *(3)* extractio *(4)* extractionem *L* 5 additione *erg.* *L* 7 repraesentent *(1)* additionem *(2)* extractionem *L* 9 involvuntur *(1)* multiplicatio *(2)* additione *L* 14 a^2x^2 *L ändert Hrsg.* 15 circuli. | Quemadmodum series inaequabiles non *gestr.* | *L*



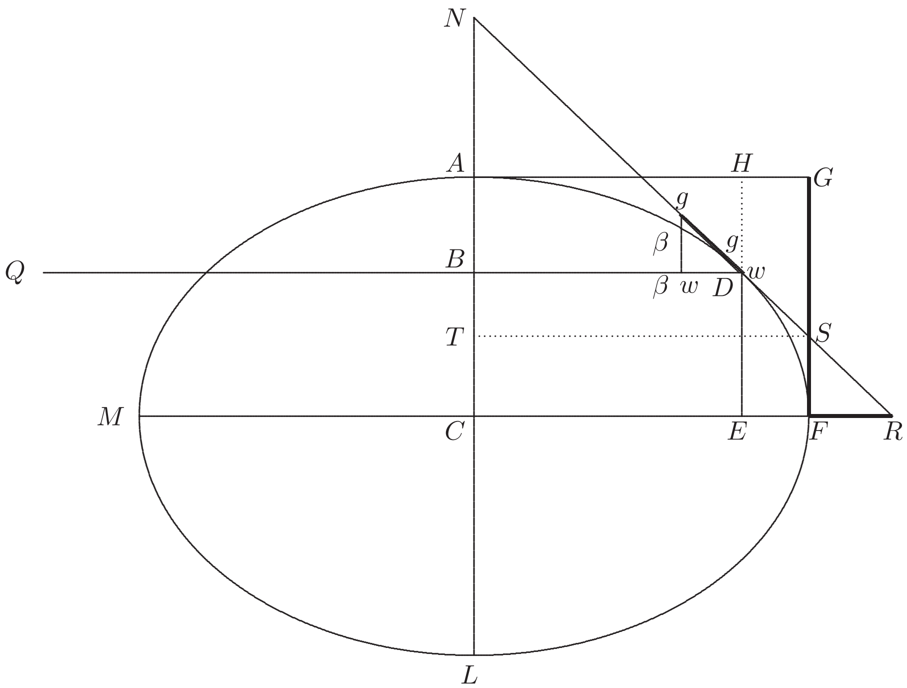
[*Fig. 9, tlw. Blindzeichnung*]

38_{15}. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS TERTIA DECIMA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 28–29. 1 Bog. 2°. 4 S. Überschrift ergänzt. Cc 2, Nr. 775 A tlw.

Pars XIII. de seriebus et summis.

[*Teil 1*]



[*Fig. 1*]

$CB \sqcap y$. $BD \sqcap x$. $AB \sqcap z$. Erit $2az - \frac{a}{b}z^2 \sqcap x^2$. ponendo $AL \sqcap 2b$. latus transversum; et $2a$ latus rectum. Pro z substituendo $b - y$ fiet $z^2 \sqcap b^2 - 2by + y^2$. et

$\frac{2ab^2-2aby-ab^2+2aby-ay^2}{b} \sqcap x^2$. sive $ab-\frac{a}{b}y^2 \sqcap x^2$, sive erit $ab^2-ay^2 \sqcap bx^2$, sive $\frac{a}{b} \sqcap \frac{x^2}{b^2-y^2}$[,] sive $b^2-y^2 \sqcap \frac{b}{a}x^2$, sive $b^2-\frac{b}{a}x^2 \sqcap y^2$, ponendo abscissam x esse CE, et applicatam y esse ED.

Quod si relatio fiat non ad axem AL. sed ad axem MF. tunc FE. appellando v et MF. appellando $2c$, et latus rectum ad eum pertinens appellando d, fiet: $2dv-\frac{d}{c}v^2 \sqcap y^2$. et pro $EF \sqcap v$, ponendo $c-x \sqcap CE$, fiet: $2dc-2dx,-\frac{d}{c}c^2+2\frac{d}{c}cx-\frac{d}{c}x^2 \sqcap y^2$. sive $\underset{II}{\boxed{2}}dc^2\boxed{-2dcx}\underset{II}{\boxed{-dc^2}}\boxed{+2dcx}-dx^2 \sqcap cy^2$, sive $c^2-x^2 \sqcap \frac{c}{d}y^2$.

Iam ut curvae ellipseos elementa investigemus: primum ex aequatione $2az-\frac{a}{b}z^2 \sqcap x^2$. quaeramus BN. posito ND. esse tangentem, nempe BN. appellata l, fiet: $\cancel{2}al-\frac{\cancel{2}a}{b}zl \sqcap \cancel{2}x^2$, sive $l \sqcap \frac{x^2}{a-\frac{a}{b}z}$. et pro x^2, substituendo eius valorem, nempe $2az-\frac{a}{b}z^2$, fiet: $\frac{2az-\frac{a}{b}z^2}{a-\frac{a}{b}z} \sqcap l$. Fiat $\frac{w}{\beta} \sqcap \frac{\sqrt{2az-\frac{a}{b}z^2}\frown a-\frac{a}{b}z}{2az-\frac{a}{b}z^2} \sqcap \frac{y}{l}$ fiet: $w \sqcap \frac{\beta,\frown a-\frac{a}{b}z}{\sqrt{2az-\frac{a}{b}z^2}}$

8 ut *(1)* curvam hype *(2)* curvae *L* 8 f. $\sqcap$ x². *(1)* et pro z. ponendo $z-\beta$. fiet: $2az-$ *(2)* quaeramus *L* 10 $l \sqcap \frac{x^2}{a-\frac{a}{b}z}$. *(1)* Et faciendo ND $\sqcap \frac{x^4}{a^2-\frac{2a}{b}z+\frac{a^2}{b^2}z^2}-$ *(2)* et *L* 11 $\sqcap$ l. *(1)* cuius quadratum est $\frac{4a^2z^2-4\frac{a^2}{b}z^3+\frac{a^2}{b^2}z^4}{a^2-\frac{2a^2}{b}z+\frac{a^2}{b^2}z^2}$ *(a)*, fiat *(b)* $\sqcap \frac{az,,+az-\frac{a}{b}z^2,,,\square}{a-\frac{a}{b}z,\ \square}$ a quo si auferatur BD^2, $\sqcap 2az-\frac{a}{b}z^2$, fiet: $\frac{az,,+az-\frac{a}{b}z^2,,,\square,,,,-az,,-az+\frac{a}{b}z^2,,,\frown a-\frac{a}{b}z,\square}{a-\frac{a}{b}z,\square}$ et liberando numeratorem a vinculis, fiet: $4a^2z^2-4\frac{a^2}{b}z^3+\frac{a^2}{b^2}z^4,,-2a^3z+\frac{4a^3}{b}z^2-\frac{2a^3}{b^2}z^3,+\frac{a^3}{b}z^2$ *(2)* Fiat *L*

cuius quadrato, $\dfrac{\beta, \frown a - \frac{a}{b}z,,\square}{2az - \frac{a}{b}z^2}$ addatur β^2, fiet: $\dfrac{\beta^2,, \frown a - \frac{a}{b}z, \square + 2az - \frac{a}{b}z^2}{2az - \frac{a}{b}z^2}$, eritque

$$\beta\sqrt{\frac{a^2 - \frac{2a^2}{b}z + \frac{a^2}{b^2}z^2 + 2az - \frac{a}{b}z^2}{2az - \frac{a}{b}z^2}} \sqcap g.$$

Quae g repraesentat momentum c u r v a e e l l i p s e o s .

Invenienda est ergo dimensio figurae, cuius elementa sint: g. et habebitur dimensio curvae ellipseos. Sed quia ea figura satis intractabilis videtur ultra pergendum est. Nimirum quaeratur curvae momentum ex AC, axe; ducantur elementa in $\sqrt{2az - \frac{a}{b}z^2}$ distantiam ab axe; fient elementa momenti curvae haec:

$$\beta\sqrt{\begin{array}{l} a^2 - \frac{2a}{b}\,z \;+ \frac{a^2}{b^2}z^2 \\ \quad + 2a\,..\; - \frac{a}{b}\,.. \end{array}}$$

quae est ad ellipsin si b maior quam a, quia tunc etiam $\frac{a}{b}$ maior quam $\frac{a^2}{b^2}$, sed ad hyperbolam, si a maior quam b. Est autem b minor quam a in nostro exemplo cum ordinata ad axem minorem applicata est, et proinde momentum curvae ex axe AL minore, est ad hyperbolam. Contra si pro a substituas d, pro b substituas c, et pro z substituas v, fiet:

$$\beta\sqrt{\begin{array}{l} d^2 - 2\frac{d}{c}v + \frac{d^2}{c^2}v^2 \\ \quad + 2d\,.. - \frac{d}{c}\,.. \end{array}}$$

quae est ad ellipsin, quia c est maior quam d; momentum ergo curvae ellipticae ADF ex axe minore AL, dependet ex quadratura hyperbolae, ex axe vero maiore dependet ex quadratura ellipseos vel circuli.

Summa omnium NS. applicatarum in BQ, aequatur rectangulo sub curva ADF, et recta CF. comprehenso. Nam $\frac{NS}{g} \sqcap \frac{TS \sqcap c}{\beta}$ ergo $NS \sqcap \frac{cg}{\beta}$. Nullus ergo novus hinc

3 f. e l l i p s e o s . *(1)* Inventa est cu *(2)* Invenienda est ergo *(a)* cu *(b)* dimensio *(aa)* curvae *(bb)* figurae *L* 8 ad *(1)* hyperbolam *(2)* ellipsin, si a ma *(3)* ellipsin *L* 10 autem *(1)* b maior quam a, si *(2)* b *(a)* maior *(b)* minor *L* 16 ADF *(1)* pendet ex axe maiore *(2)* ex *L* 18 BQ, *(1)* ducta in CF, *(2)* aequatur *L*

ducitur calculus, porro g relata ad axem maiorem MR, eundem habet valorem, quem ad axem minorem, nisi quod ut dixi, pro a. b. z. aliae quas dixi lineae, d. c. v. substituendae sunt.

Nunc eadem aequatione $2az - \frac{a}{b}z^2 \sqcap x^2$, inverse utamur; nempe sumta AH pro abscissa[,] $DH \sqcap z$ pro ordinata, et quaeramus $D\lambda$ pro intervallo tangentis quam vocabimus λ nempe fiet $\not{2}az - \frac{\not{2}a}{b}z^2 \sqcap \not{2}x\lambda$ et $\lambda \sqcap \frac{az - \frac{a}{b}z^2}{x}$, huius aequationis priori iunctae ope tollamus terminum z. Nempe primum z^2, pro $-\frac{a}{b}z^2$, substituendo in posteriore eius valorem ex priore, qui est $x^2 - 2az$, et fiet: $\boxed{az} + x^2 - \boxed{2}az \sqcap x\lambda$. et erit $z \sqcap \frac{x^2 - x\lambda}{a}$, et $z^2 \sqcap \frac{x^4 - 2x^3\lambda + x^2\lambda^2}{a^2}$, quibus valoribus in prima aequatione substitutis, fiet:

$$\frac{2a^{\not{2}}x^{\not{2}} - 2a^{\not{2}}\not{x}\lambda - \frac{\not{a}}{b}x^{\not{4}3} + \frac{2\not{a}}{b}x^{\not{3}2}\lambda - \frac{\not{a}}{b}x^{\not{2}}\lambda^2}{a^{\not{2}}} \sqcap x^{\not{2}},$$ qua aequatione exprimitur valor ipsius λ aequatione tali: $2a[x]\ -2a\lambda - \frac{x^3}{b} + \frac{2x^2\lambda}{b} - \frac{x\lambda^2}{b} \sqcap ax$.

Sed brevius forte ita rem obtinebimus: $z^2 - 2bz + b^2 \sqcap b^2 - x^2$, et $\mp z \pm b \sqcap \sqrt{b^2 - x^2}$, et $z \sqcap \mp\sqrt{b^2 - x^2} + b$. Pro x substituendo $x + \beta$. fiet: proxima $z \sqcap \mp\sqrt{b^2 - x^2 - 2x\beta - \beta^2} + b$. Auferatur illa ab hac, fiet: $\mp\sqrt{b^2 - x^2 - 2x\beta - \beta^2} \pm \sqrt{b^2 - x^2} \sqcap (w)$. Unde

$$+b^2 - x^2 - 2x\beta - \not{\beta^2} - 2\sqrt{b^2 - x^2 - 2x\beta \not{-\beta^2}, \frown b^2 - x^2} + b^2 - x^2 \sqcap w^2.$$

$$2b^2 - 2x^2 - 2x\beta\ -2\sqrt{\ldots} \sqcap w^2, \text{ sive :}$$

$$2b^2 - 2x^2 - 2x\beta\ -w^2 \quad \sqcap 2\sqrt{\ldots}, \text{ et quadrando utrobique}$$

$$\boxed{4b^4}\boxed{-8b^2x^2}\ -8b^2x\beta\ -4b^2w^2\boxed{+4x^4} + 8x^3\beta + 4x^2w^2, +4x^2\beta^2 + 4x\beta w^2 + w^4 \sqcap$$
$$\boxed{4b^4}\boxed{-8b^2x^2}\boxed{+4x^4} - 8b^2\beta x + 8x^3\beta.$$

6 $\lambda \sqcap \frac{az - \frac{a}{b}z^2}{x}$, *(1)* sive $\lambda^2 \sqcap$ *(2)* sive λ *(3)* huius *L*   11 x^2 *L ändert Hrsg.*

11 f. $\sqcap$ ax. *(1)* Ita efficia *(2)* Sed *L*   14 f. Unde *(1)* $\boxed{+b^2 - x^2} - 2x\beta - \not{\beta^2} - 2\sqrt{b^2 - x^2 - 2x\beta - \not{\beta^2}, \frown b^2 - x^2}\boxed{+b^2 - x^2} \sqcap w^2$. sive $w^2 + 2x\beta, \square \sqcap b^2 - x^2 - 2x\beta, \frown b^2 - x^2$, sive: $w^4 + 4x\beta w^2 + 4x^2\beta^2 \sqcap b^4 - 2$ *(2)* $+b^2$ *L*

Quod si iam ad w^2 addatur β^2, radix summae repraesentabit etiam elementa ellipseos, sed haec quoque compositiora quam velim. Ope tamen ipsius λ facilius habetur exitus, quia ibi λ habetur pura.

Satius ergo erit ad reliquas progredi aequationes. Sumamus scilicet aequationem: $b^2 - \frac{b}{a}x^2 \sqcap y^2$. et quaeramus $ER \sqcap \daleth$. ponendo $CE \sqcap x$ pro ordinata; et $ED \sqcap y$ pro abscissa nempe fiet:

$$-\frac{\cancel{2}b}{a}\daleth \sqcap \cancel{2}y^2, \text{ et } \daleth \sqcap -\frac{b^2 - \frac{b}{a}x^2}{\frac{b}{a}x}, \text{ fiat } \frac{w}{\beta} \sqcap \frac{\sqrt{b^2 - \frac{b}{a}x^2}}{b^2 - \frac{b}{a}x^2} \frown \frac{b}{a}x, \text{ sive } w \sqcap \frac{\beta\frac{b}{a}x}{\sqrt{b^2 - \frac{b}{a}x^2}}. \text{ eius}$$

quadrato $\frac{\beta^2\frac{b^2}{a^2}x^2}{b^2 - \frac{b}{a}x^2}$, addatur β^2, producti radix fiet: $\beta\sqrt{\frac{\cancel{\beta^2}\frac{b^2}{a^2}x^2 + b^2\cancel{\beta^2} - \frac{b\beta^2}{a}x^2}{b^2 - \frac{b}{a}x^2}} \sqcap g$.

Superest ergo ut huius figurae dimensionem tentemus.

Nimirum aequatio fit: $\frac{b^2g^2 - \frac{b}{a}x^2g^2}{\beta^2} \sqcap \frac{b^2}{a^2}x^2 + b^2 - \frac{b}{a}x^2$.

$$\boxed{-\frac{b}{a}\,..}$$

Huius figurae g^2 habentur ex quadratura hyperbolae: fit enim $g^2 \sqcap \frac{\frac{b^2}{a^2}x^2 + b^2 - \frac{b}{a}x^2}{b^2 - \frac{b}{a}x^2}$

$\sqcap \frac{\frac{b^2}{a^2}x^2}{b^2 - \frac{b}{a}x^2} + 1$. quae pendent ex quad. hyp. At x^2, fit $\sqcap \frac{b^2g^2}{\underbrace{\frac{b^2}{a^2} - \frac{b}{a}} + \frac{b}{a}g^2} - \frac{[b^2]}{\frac{b^2}{a^2} - \frac{b}{a} + \frac{b^2}{a^2}g^2}$

quae pendent ex quad. vel circuli si $\underbrace{\frac{b^2}{a^2} - \frac{b}{a}}$ est quantitas affirmativa, vel hyperbolae

9 huius *(1)* curvae *(2)* figurae *L* 11 *Umrahmung erg. Hrsg.* 12 figurae *(1)* momenta g^2 habentur ex quadratura hyperbolae. Eiusdem x^2, etiam habentur ex quadratura hyperbolae *(2)* g^2 *L* 13 $\frac{b}{a}g^2$ *L ändert Hrsg.*

si negativa. Duo ergo momenta habemus figurae ipsarum g. Ducantur g in $b+x\sqrt{\dfrac{b}{a}}$ fiet: momentum omnium g ex puncto versus verticem, symmetrum summae ipsarum x^2, id tanquam fiet: $\sqrt{\dfrac{\dfrac{b^2}{a^2}x^2,, + b + x\sqrt{\dfrac{b}{a}} \frown b - x\sqrt{\dfrac{b}{a}},,, \frown b + x\sqrt{\dfrac{b}{a}}}{b - x\sqrt{\dfrac{b}{a}}}}$. Curvae hyperbolicae homogenea est haec figura: $\dfrac{\sqrt{a^4+y^4}}{y^2}$. Item haec $\sqrt{\dfrac{2x^2 ǂ\, a^2}{ǂ a^2 + x^2}}$. quae posterior similis facile reddi potest homogeneae curvae ellipticae dividendo per $\sqrt{\dfrac{b}{a}}$. cognitam: Elementa curvae hyperbolicae ex posteriore etiam sic enuntiari possunt: $\sqrt{1+\dfrac{x^2}{ǂ a^2+x^2}}$. Utamur si licet extractione per partes: $\sqrt{1+\dfrac{x^2}{ǂ a^2+x^2}} \sqcap 1+z$ ponendo $2z+z^2 \sqcap \dfrac{x^2}{ǂ a^2+x^2}$. Sed ut progressio fiat in infinitum, ita procedemus:

$$\sqrt{\cancel{1} \quad + \quad \dfrac{\cancel{x^2}}{\cancel{ǂ a^2 + x^2}} + \dfrac{\cancel{x^4}}{\cancel{ǂ a^2 + x^2,\square}} - \dfrac{x^4}{ǂ a^2 + x^2,\square}}$$

$$1 \qquad \dfrac{x^2}{ǂ a^2 + x^2,, \frown 2} \qquad \dfrac{-x^4}{ǂ a^2+x^2,\cancel{\square}} \smile \dfrac{ǂ 2a^2 + 3x^2}{\cancel{ǂ a^2 + x^2}}$$

$$\cancel{1} \qquad \dfrac{\cancel{x^2}}{\cancel{ǂ a^2 + x^2}} + \dfrac{\cancel{x^4}}{\cancel{ǂ a^2 + x^2,\square}}$$

$$\dfrac{ǂ 2a^2 + 2x^2 + x^2}{ǂ a^2 + x^2}$$

1 ergo *(1)* | curvae *streicht Hrsg.* | *(a)* huius momenta habemus *(b)* momenta habemus ipsarum g *(2)* momenta *L* 2 ex *(1)* verti *(2)* puncto *(a)* ultra verticem *(b)* versus *L* 8 fiat *(1)* in infinitum faciemus ita: $\sqrt{1+\dfrac{x^2}{ǂ a^2+x^2}} \sqcap 1+$ *(2)* in *L*

9 Bei der quadratischen Ergänzung unter der Wurzel vergißt Leibniz den Faktor 4 im Nenner der letzten zwei Terme, was die Berechnung der weiteren Terme des Schemas beeinträchtigt.

$$\cancel{a^2} + \cancel{bc} + \cancel{\frac{b^2c^2}{4a^2}} - \cancel{\frac{b^2c^2}{4a^2}} + \cancel{\frac{b^4c^2}{2a^2+bc,4,,\square}} - \frac{b^4c^2}{2a^2+bc,4,\square}$$

$$a \;\Big\|\; \frac{bc}{2a} \qquad \frac{-b^2c^2}{2a^2+bc,,4} \qquad \frac{-b^4c^2}{2a^2+bc,\square,4,,\frown 2a^2+bc,4,-b^2c^2} \quad \text{etc.} \;\sqcap\; \sqrt{a^2+bc}$$

$$a \qquad \frac{bc}{2a} \qquad \frac{2a+bc}{2a}\Big) \qquad \frac{2a^2+bc,,\frown 2a^2+bc,4,-b^2c^2}{2a^2+bc,4}\Big)$$

$$bc + \frac{b^2c^2}{4a^2} - \cancel{\frac{b^2c^2}{4a^2}} + \cancel{\frac{b^4c^2}{2a^2+bc,4,\square}}$$

Regula continuandi haec est, ut numerator termini sequentis sit quadratum a numeratore proxime praecedentis, nominator autem eius sit numerator fractionis factae ex praecedentibus terminis omnibus ad commune nomen redactis, ductus in nominatorem ultimi.

Esto radix extrahenda ex 2. seu ex $\sqrt{1+1}$, fiet:

10–520,2 *Nebenrechnungen:*

$$\frac{3}{2} - \frac{1}{12} \sqcap \frac{18-1}{12} \qquad \begin{array}{r} 17 \\ 12 \\ \hline 34 \\ 17 \\ \hline 204 \end{array}$$

$$204 \begin{cases} 2 \\ \\ 102 \end{cases} \qquad 12 \left\{ \begin{array}{c} 2 \\ \\ 6 \end{array} \right\} 12 \qquad \begin{cases} 6 \\ 17 \end{cases}$$

1–5 In den Zählern der letzten zwei Terme der ersten Zeile des Schemas müßte b^4c^4 stehen, im Nenner des dritten Terms der zweiten Zeile fehlt der Faktor a; beides beeinträchtigt die Berechnung der weiteren Terme. 8 f. ductus in nominatorem ultimi: Leibniz vergißt, mit dem Faktor 2 zu multiplizieren. Vgl. N. 32 S. 347 Z. 3–7. 10 $\sqrt{1+1}$: Leibniz rechnet ab dem dritten Term der Reihendarstellung mit der fehlerhaften Regel, ein Schreibfehler in einer Nebenrechnung verzerrt das Ergebnis zusätzlich.

$\frac{3}{2} - \frac{1}{12} - \frac{1}{204} - \frac{1}{22432} - \frac{1}{167611904}$ etc. Ergo ratio diagonalis ad latus in quadrato, est ut $\frac{3}{2} - \frac{1}{12} - \frac{1}{204} - \frac{1}{22432} - \frac{1}{167611904}$ etc. ad 1.

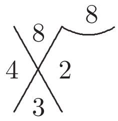
Eodem modo per extractionem radicis cubicae de 2. in infinitum continuatam, exprimi potest ratio lateris cubi dupli ad latus cubi dati; id est duplicatio cubi.

Quod si vero iam quaelibet harum fractionum ex quibus componitur numerus irrationalis, resolvatur in integros, patet in infinitas paraboloeides resolvi posse figuram quamlibet. Eadem etiam ad radicum altiorum radices applicari possunt; addo id servire ad radices cognitas etiam ex affectis extrahendas.

Sed superest alia quoque ratio resolvendi radicem in infinitas alias quantitates, sed in quas ni fallor ipsa radix ingreditur:

17	102		208 [!]	208	11216
6	3		204	17	3536
102	306	−17 − 1, ⌢ 204	832	1456	208
	17		2160	208	3744
	289		22432	3536	7472
	1				
	288				

22432
7472
44864
157024
89728
157024
167611904



6 f. cubi. *(1)* Sed aliam *(2)* Quod *L* 8 posse *(1)* quantitatem *(2)* figuram *L*

Ut esto: $\sqrt{a^2 \pm bc} \sqcap a \pm z$. Erit $\boxed{a^2} \pm 2az + z^2 \sqcap \boxed{a^2} \pm bc$; et pro z substituendo eius valorem, fiet: $z^2 \sqcap \pm\, bc - 2a\sqrt{a^2[\pm]bc} + 2a^2$ et $z \sqcap \sqrt{\pm\, bc - 2a\sqrt{a^2[\pm]bc} + 2a^2}$. Nam $z \sqcap \pm\sqrt{a^2 \pm bc} \mp a$, ex hypothesi. Ergo $\pm\, 2az \sqcap 2a\sqrt{a^2 \pm bc} - 2a^2$. Ergo $z^2 \sqcap \pm\, bc - 2a\sqrt{a^2 \pm bc} + 2a^2$. Et $z \sqcap \sqrt{2a^2 \pm bc - 2a\sqrt{a^2 \pm bc}}$. Et $\sqrt{a^2 \pm bc} \sqcap a \pm \sqrt{2a^2 \pm bc - 2a\sqrt{a^2 \pm bc}}$.

Haec methodus duo habet praeclara supra priorem; nam neque fractos introducit, ubi non iam tum adsunt, et quod superior non potest, finiri potest ubilibet, et servit ad alias figuras ad alias reducendas; ponendo $\pm \sqcap -$ et $bc \sqcap x^2$ fiet $+\sqrt{2a^2 - x^2 - 2a\sqrt{a^2 - x^2}} \sqcap -y + a$, et fiet quadrando:

$\boxed{2}a^2 - 2a\sqrt{a^2 - x^2} \sqcap y^2 + 2ya\boxed{+a^2}$. Unde $2a\sqrt{a^2 - x^2} \sqcap a^2 - y^2 - 2ya$. et rursus quadrando: $3\boxed{4}a^4 - 4a^2x^2 \sqcap \boxed{a^4}\ \boxed{-2a^2y^2} - 4a^3y + y^4 + 4y^3a + 2\boxed{4}y^2a^2$. Sed calculo errorem inesse iudico, nam in $2a\sqrt{a^2 - x^2} \sqcap a^2 - y^2 - 2ya$ substituendo pro $\sqrt{a^2 - x^2}$ eius valorem y, fit y aequalis quantitati determinatae, quod est absurdum. Error ergo in calculo, resumamus.

$\sqrt{a^2 - x^2} \sqcap a - z$. Ergo $\boxed{a^2} - x^2 \sqcap \boxed{a^2} - 2az + z^2$. Iam ex hypothesi $z \sqcap a - \sqrt{a^2 - x^2}$. Ergo $-x^2 \sqcap -2a^2 + 2a\sqrt{a^2 - x^2} + z^2$. sive $z \sqcap \sqrt{+2a^2 - x^2 - 2a\sqrt{a^2 - x^2}}$ ut ante $\sqcap a - \sqrt{a^2 - x^2}$. Unde $\sqrt{a^2 - x^2} \sqcap a - \sqrt{2a^2 - x^2 - 2a\sqrt{a^2 - x^2}}$. Et videndum est an radix haec posterior possit esse impossibilis seu imaginaria, etsi prior sit realis, nimirum ponamus $a \sqcap x + e$. et $2a^2 + f \sqcap x^2 + 2a\sqrt{a^2 - x^2}$. Videamus an non ex hac hypothesi sequatur absurdum. Pro x pone $a - e$. Unde $\boxed{2}a^2 + f \sqcap \boxed{a^2} - 2ae + e^2 + 2a\sqrt{\boxed{a^2 - a^2} + 2ae - e^2}$.

2 $z^2 \sqcap$ *(1)* $bc \pm 2a\sqrt{a^2 + bc}$ et $z \sqcap \sqrt{bc \mp 2a\sqrt{a^2 + bc}}$. | Ergo *(a)* $\sqrt{a^2 + bc} \sqcap a \boxed{+\sqrt{bc \mp 2a\sqrt{a^2 + bc}}}$ *(b)* $\sqrt{a^2 \pm bc} \sqcap a \pm \underbrace{\sqrt{bc \mp 2a\sqrt{a^2 + bc}}}_{\sqrt{bc} + \sqrt{2a\sqrt{\ }}}$ *streicht Hrsg.* | *(2)* $\pm bc -$ | $2a\sqrt{a^2 + bc} + 2a^2$ et $z \sqcap \sqrt{\mp bc - 2a\sqrt{a^2 + bc} + 2a^2}$. *ändert Hrsg.* | *(a)* Nam $z \sqcap \pm\sqrt{a^2 \pm bc} \mp a \sqcap \sqrt{\pm}$ *(b)* Nam *L* 5 priorem; *(1)* etsi enim in eo inferior *(2)* nam *L* 15 f. $\sqcap a - \sqrt{a^2 - x^2}$ *(1)*, quod statim probari potest nam *(2)*. Unde *L*

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8 fiet quadrando: Auf der linken Seite der Gleichung fehlt der Term $-x^2$, auf der rechten müßte $-2ya$ stehen. Leibniz bemerkt die Unstimmigkeit und setzt neu an.

et transponendo et $a^2 + f + 2ae - e^2$ ducendo in se fiet: $a^4 + 2a^2f + 4a^3e - 2a^2e^2, +f^2 + 4aef - 2fe^2, +4a^2e^2 - 4ae^3 + e^4 \sqcap 8a^3e - 4a^2e^2$. Sed hoc prolixius, manifestum aliunde est radicem eam fieri posse imaginariam, $f \sqcap x^2 + 2a\sqrt{a^2 - x^2} - 2a^2$. Optime id patebit, ponendo x talem ut radix extrahi possit ex quadrato $a^2 - x^2$. Pone $x \sqcap 3$. $a \sqcap 5$. fiet $f \sqcap 9 + 40 - 50$. Ergo $f \sqcap -1$. eoque casu radix realis manet: $a^2 - x^2 \sqcap d^2$, pone $a \sqcap x + g$. fiet $\boxed{x^2} + 2gx + g^2 \boxed{-x^2} \sqcap d^2$. et $\dfrac{d^2 - g^2}{2g} \sqcap x$. Pone $d \sqcap hg$, erit $x \sqcap \dfrac{h^2g - g}{2}$, et pro g ponendo $2m$ fiet: $x \sqcap h^2m - m$. $d \sqcap 2hm$. et $a \sqcap h^2m \boxed{-m} + \textcircled{2}m$.

Pone $h \sqcap 2$ et $m \sqcap 10$, fiet $a \sqcap 50$. et $x \sqcap 30$, et $a^2 - x^2 \sqcap 2500 - 900$ seu $\begin{array}{r} 2500. \\ 900 \\ \hline 1600 \end{array}$

Ponendo $h \sqcap 10$ et $m \sqcap 10$ fiet $a \sqcap 1000 + 10$. et $a^2 \sqcap$ $\begin{array}{r} 100010 \\ 100010 \\ \hline 1000100 \\ 100010000 \\ \hline 10002000100 \end{array}$ [!].

$h \sqcap 3$ fiet: $90 + 10 \sqcap 100 \sqcap a$. et $x \sqcap 80$. et $a^2 - x^2 \sqcap 10000 - 6400 \sqcap$ $\begin{array}{r} 10000. \\ 6400 \\ \hline 3600 \end{array}$ Unde

$f \sqcap 6400 + \underbrace{2 \frown 100 \frown 60}_{12000} - 20000$.

Pone $h \sqcap \dfrac{1}{2}$. fiet $a \sqcap \dfrac{10 + 40}{4}$. et $x \sqcap \dfrac{-30}{4}$. fiet $2500 - 900 \sqcap 1600$.

In aequatione $f \sqcap x^2 + 2a\sqrt{a^2 - x^2} - 2a^2$ substituendo valores inventos fiet:

$\boxed{h^4m^2}\boxed{-2h^2m^2}\boxed{+m^2} + \underbrace{2h^2m + 2m,, \frown 2hm,}_{4h^3m^2 + 4hm^2} -\textcircled{2}h^4m^2 - \left[6\textcircled{4}\right] h^2m^2 - \textcircled{2}m^2$ et

$\dfrac{f}{m^2} \sqcap 4h^3 + 4h - h^4 - 6h^2 - 1$.

8 Pone *(1)* h ⊓ 10, et m ⊓ 100, fiet: a ⊓ 100000 *(!)* +100 ⊓ 100100 et $a^2 \sqcap$ $\begin{array}{r} 10010000 \\ 1001 \\ \hline 10010000 \\ 100100 \\ \hline 10020010000 \end{array}$ *(2)* h ⊓ 2 *L*

22 4$\textcircled{2}$ *L ändert Hrsg.*

Nam si $x \sqcap 3.$ et $a \sqcap 5.$ erit $g \sqcap 2.$ et $m \sqcap 1.$ et $h \sqcap 2.$ Ita enim $\dfrac{h^2g-g}{2} \sqcap 3.$ et $h^2g+g \sqcap 5.$

$$\begin{array}{ccccccccc} 4h^3 & + & 4h & - & h^4 & - & [6]h^2 & & \\ & & & & & & & -1 & \\ 32 & + & 8 & - & 16 & - & [24] & & \end{array}$$

$$\begin{array}{cccccccc} \boxed{h^4m^2} & \boxed{-2h^2m^2} & \boxed{+m^2} & +4h^3m^2 + & 4hm^2 & -\textcircled{2}h^4m^2 - & 6\textcircled{4}h^2m^2 & -\textcircled{2}m^2 \\ 16 & -8 & +1 & +32 & +8 & -32 & -16 & -2 \\ & & & \underbrace{\qquad\qquad\qquad}_{40} & & \underbrace{\qquad\qquad\qquad\qquad\qquad}_{-50} & & \\ \hline & & & +32 \; + & 8 & -16 & -24 & -1 \\ & & & \underbrace{\qquad\qquad\qquad}_{40} & & \underbrace{\qquad\qquad\qquad\qquad\qquad}_{-40-1} & & \\ & & & & \underbrace{\qquad\qquad\qquad\qquad\qquad}_{-\;1} & & & \end{array}$$

Restat ergo examinandum an ita possibile sit f esse quantitatem affirmativam, nam si potest, radix realis imaginarie exprimi potest, instar cubicarum Cardani radicum ageometricarum.

Sed iam video id fieri non posse, quia mutatis omnibus signis et ordinando, fiet $-f \sqcap h^4-4h^3+6h^2-4h+1.$ quae est quantitas quadrato quadratica ab $h-1$, ac proinde non potest esse negativa, nec proinde f affirmativa. Itaque de radicibus imaginariis aliter exprimendis parum hinc spero.

Valde notabile est ex progressione aliqua in infinitum continuata non semper sequi seriem esse termino ex quo fit aequalem; nisi demonstrari possit in infinitum continuando ultimum fore aequalem nihilo. Idque patet in posteriore meo extrahendi modo; ubi patet continuo radicem extrahi posse, ita ut extractas partes non ingrediatur bc; sed tantum a.

Generaliter quandocunque datur appropinquatio terminorum in infinitum accedentium; datur series infinita aequalis quantitati quaesitae. Nimirum sumendo omnium ter-

3 4 *L ändert Hrsg.* 5 16 *L ändert Hrsg.* 13–15 radicum *(1)* non *(2)* ageometricarum. *(a)* Scilicet: fiet *(b)* Sed *L*

22 non ingrediatur bc: Leibniz berücksichtigt nicht, daß er in S. 521 Z. 7 bc durch x^2 ersetzt hat.

minorum accedentium differentias; eorum summa si quidem termini accedentes crescunt (ut polygona inscripta v. g.) est aequalis termino quaesito. At si decrescunt summa eorum aequatur excessui termini maximi super ignotam quantitatem.

Si recta secatur in extrema et media ratione observatum est, a quibusdam, hanc seriem seu rationem numerorum,

$$1 \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{5} \quad \frac{5}{8} \quad \frac{8}{13}$$

continue accedere ad rationem minoris termini ad maiorem.

Ego addo eius diff.

$$\frac{1}{2} \quad -\frac{1}{6} \quad +\frac{1}{15} \quad -\frac{1}{40} \quad +\frac{1}{104}$$

quarum habetur summa aequalis rationi portioni minoris ad maiorem.

[*Teil 2*]

$$\mp\ \frac{\frac{b}{a}y^2+cy+da}{\frac{e}{a}y^2+fy+ga}\ \ddagger\ \frac{\begin{array}{l}\frac{b}{a}y^2 \quad +cy \quad +da\\ \frac{2b}{a}y\beta \ +c\beta\\ \frac{b}{a}\beta^2\end{array}}{\begin{array}{l}\frac{e}{a}y^2 \quad +fy+ga\\ \frac{2e}{a}y\beta \ +f\beta\\ \frac{e}{a}\beta^2\end{array}}\quad \sqcap$$

10 *Nach* maiorem: [illegible]

3 f. quantitatem. *(1)* Differentiae autem *(2)* Si *L*

4 a quibusdam: z. B. von J. Kepler, *Strena*, 1611, S. 12 (*KW* IV S. 270) u. *Harmonice mundi*, 1619, S. 76 f. u. 183 (*KW* VI S. 175 f. u. 294) sowie A. Girard in S. Stevin, *L'arithmétique*, 1634, S. 169 f.

10 portioni: Die Partialsummen der Differenzenreihe ergeben die Folge $\frac{1}{2}, \frac{1}{3}, \frac{2}{5}, \frac{3}{8}, \frac{5}{13}$ etc.

$$\frac{eb}{a^2}y^4 \quad \frac{2eb}{a^2}\beta y^3 \quad \frac{eb}{a^2}\beta^2 y^2$$
$$\frac{ec}{a}\,.. \mp \frac{2ec}{a}\beta\,.. \mp \frac{ec}{a}\beta^2 y$$
$$\frac{ed\phi}{\phi}\,.. \mp \frac{2eda}{a}\beta\,.. \mp \frac{eda}{a}\beta^2$$
$$\frac{fb}{a}\,.. \quad \frac{fb}{a}\beta\,..$$
$$cf\,.. \quad fc\beta\,..$$
$$fda\,.. \mp fda\beta$$
$$\frac{gab}{a}\,.. \quad gac\,.. \quad gda^2$$

divisa per

$$\frac{eb}{a^2}\,.. \quad \frac{2eb}{a^2}\beta\,.. \quad \frac{eb}{a^2}\beta^2\,..$$
$$\frac{ec}{a}\,.. \quad \frac{ec}{a}\beta\,..$$
$$\frac{eda}{a}\,..$$
$$\frac{fb}{a}\,.. \pm \frac{2fb}{a}\beta\,.. \pm \frac{fb}{a}\beta^2\,..$$
$$fc\,.. \quad fc\beta\,..$$
$$fda\,..$$
$$\frac{gab}{a}\,.. \pm \frac{2gab}{a}\beta\,.. \pm \frac{gab}{a}\beta^2$$
$$gac\,.. \pm gac\beta$$
$$gda^2$$

1–526,9 NB. In hac fractione y^2 et y tolli possunt ex numeratore ut inde fiat fractio cuius [numerator] constans et restabunt in nominatore quantitates arbitrariae; quarum ope videndum est an nominator possit accipere formam quadrato-quadrati aut etiam quadrati.

17 hac *(1)* serie *(2)* fractione *L* 17 ex *(1)* denomi *(2)* numeratore *L* 18 nominator *L ändert Hrsg.*

$$
\begin{array}{llllll}
\frac{e^2}{a^2}y^4 + \frac{2e^2}{a^2}\beta\, y^3 + \frac{e^2}{a^2}\beta^2 & y^2 & & & & \\
\quad + \frac{ef}{a} \quad .. + \frac{ef}{a}\beta & .. & & & & \\
\qquad\quad + \frac{ega}{a} & .. & & & & \\
\qquad \frac{fe}{a} \quad .. + \frac{2fe}{a}\beta & .. & + \frac{fe}{a}\beta^2 & y & & \\
\qquad\qquad f^2 & .. & f^2\beta & .. & & \\
 & & fga & .. & & \\
\qquad\qquad \frac{gae}{a} & .. & \frac{2gae}{a}\beta & .. & \frac{gae}{a}\beta^2 & \\
 & & gaf & .. & gaf\beta & \\
 & & & & g^2a^2 &
\end{array}
$$

Si ex producto facere velis figuras geometricas, reice in numeratore omnia multiplicata per β^2, et in nominatore omnia in quibus est β, fiet: $\dfrac{\begin{array}{l} \text{ǂ}\,\frac{ec}{a}y^2 \;\text{ǂ}\, 2edy \;\text{ǂ}\, fda \\ \ddagger\,\frac{fb}{a}.. \;\ddagger\, 2gb.. \;\ddagger\, gac \end{array}}{\frac{e}{a}y^2 + fy + ga, \square}$. Et proinde omnes figurae quae ordinatam habent valoris puri rationalis, qui ad hunc reduci potest, sunt quadrabiles. Hinc data aliqua serie, cuius terminus est valoris puri rationalis, sed in qua abscissa y ingreditur denominatorem, ille denominator multiplicandus est per proxime maiorem, multiplicando totam fractionem per ipsam. Inde numerator cogitandus est compositus ex duabus partibus; altera multipla nominatoris initio dati, altera multipla proxime maioris. Multipla inquam per aliam quantitatem, ita ut ea per quam multipla est nominatoris sit proxime maior ea per quam multipla est proxime maioris [numerator] quantitatis.

Pro figuris geometricis, fractio ordinatae valorem exhibens multiplicetur per denominatorem, denominator fiet quadratus. Si opus est videbitur adhuc fractio per aliam

14 aliqua *(1)* figura, cuius ordinata *(2)* serie *L* 15 denominator, *(1)* radicandus est quadratus *(2)* multiplicandus *L* 17 f. multipla *(1)* radicis *(2)* nominatoris | initio dati *erg.* | altera *(a)* multiplum eiusdem radicis sed *(b)* multipla proxime maioris. *(aa)* Sed ita ut quantitas multipla ra *(bb)* Multipla *L* 19 est *(1)* radix *(2)* nominatoris *L* 19 f. est proxime *(1)* maior radix *(2)* maioris | nominatore *ändert Hrsg.* | quantitatis *L*

quantitatem quadratam multiplicari posse. Ac postea numerator quoque resolvendus. Sed ut hoc fiat facilius utile erit, continuatis eiusmodi tabulis constantem quendam progressum notare, tum ut in altioribus non sit opus calculatione tabulae, sed ut scribi statim possit; tum ut inde lux ad regressum habeatur. Praeterea, crediderim ita formula data tandem iudicari posse an eam aptari tali seriei possibile sit, an vero id sit impossibile, utcunque per formulas arbitrarias q u a d r a t a s multiplicetur. In numeris semper pro quadratis formulis sunt quasi triangulares, quantitatis in proxime maiorem.

Unum restat indagandum an possibile sit inveniri series inaequabiles, quarum differentiae sint aequabiles. Sunt autem inaequabiles aliae, quia in ipsis crescunt exponentes, aliae, quia non ex numero solo naturali componi possunt. Credibile est figuras geometricas quae non sunt quadrabiles per aequabilem, ut figura angulorum, esse per inaequabilem, ratione exponentium ut puto, sed aequabilitati mixtam. Figurae logarithmorum differentiae sunt applicatae hyperbolae.

5 an *(1)* in infinitum *(2)* eam *L*

38_{16}. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS QUARTA DECIMA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 30–31. 1 Bog. 2°. 4 S. Isolierte Rechnung auf Bl. 31 v° rechts unten, schräg geschrieben (= S. 538 Z. 12–16) — Berechnung der Größen e, g, h, l auf LH 35 XII 2 Bl. 62 r° (Cc 2, Nr. 00 = S. 537 Z. 12–14). 4 Z. Auf dem Rest des Blattes Cc 2, Nr. 543 (Druck in einem späteren Band der Ausgabe).
Cc 2, Nr. 775 A tlw., 00.

Pars XIV schediasmatis de seriebus et summis.

Si in formula sub finem tabulae praecedentis ponas e, et b nihilo aequales fiet $\dfrac{[\ddagger] fda \ddagger gac}{f^2y^2 + \; f^2\beta y \; + \; gaf\beta}$. Si iam nominatorem huius formulae cum ista conferas:
$\quad + 2fga.. \; + \; g^2a^2$

$f^2y^2 + 2mafy + m^2a^2$ fiet: $\beta f + 2ga \sqcap 2ma$, sive $m \sqcap \dfrac{\beta f + 2ga}{2a}$. Ergo quia $a^2m^2 \sqcap gaf\beta + g^2a^2$ fiet: $\dfrac{\beta^{\not 2} f^{\not 2} + 4\not\beta \not f ga + 4g^2a^2}{4} \sqcap ga\not f\beta \boxed{+g^2a^2}$, vel $\beta f \sqcap 0$.

Quod cum sit contra propositum, quia f posita $\sqcap 0$. in nominatore restabit tantum g^2a^2, contra propositum, quia quaerimus quantitatem variantem. At β poni $\sqcap$ 0. non licet, quia differentia alioquin quae quaeritur erit infinite parva seu nulla, quod non nisi in geometricis ordinatarum seriebus locum habet.

Hinc colligo seriem ipsarum $\dfrac{1}{y^2}$ summatricem non posse habere terminorum sive ordinatarum valorem purum, in qua y, ultra y^2 non ascendat in nominatore. Quare necesse esse ut ascendat altius; sed ut postea divisione etc. deprimatur differentia ad habendas $\dfrac{1}{y^2}$.

Videamus iam an quadrata unitate minuta, per saltum assumta, quae ita exprimi potest: $\dfrac{1}{z \frown, z + \frac{1}{2}\beta}$. ponendo β esse per quam crescunt ipsae z. Quod si ponatur β aequale

10 $\ddagger$ *erg. Hrsg.*

9 formula: N. 38_{15} S. 526 Z. 11 f.

ipsi 4 erit series circuli quadratrix. Quam formulam, ut conferamus superiori, dividamus nominatorem superioris per f^2, fiet: $y^2 \left\{ \frac{+f^{\cancel{2}}\beta y \;\; +2\cancel{f}ga..}{f^{\cancel{2}}} \right. \left\{ \frac{+gaf\beta \;\; +g^2a^2}{f^2} \right.$ et conferenda: $z^2 + \frac{1}{2}\beta z$. explicando y, fiet $y \sqcap z + d$. et fiet:

$$\overbrace{\underbrace{\sqcap \quad z^2 \left\{ \frac{+2df\; z \;\; +f\beta\; .. \;\; +2ga\; ..}{f} \right. \left\{ \frac{+d^2f^2 \;\; +f^2\beta d \;\; +2fgad \;\; +gaf\beta \;\; +g^2a^2}{f^2} \right.}_{\sqcap}}^{z^2+\frac{1}{2}\beta z \qquad *}$$

Iam si $\frac{2df + f\beta + 2ga}{f} \sqcap \frac{1}{2}\beta$. erit $4df + \boxed{2}f\beta + 4ga \sqcap \boxed{f\beta}$. ac proinde $4df + f\beta + 4ga \sqcap 0$. et $g \sqcap \frac{-4df - f\beta}{4a}$. et $g^2 \sqcap \frac{4d^2f^2 + 8f^2d\beta + f^2\beta^2}{16a^2}$. et inserendo hos valores,

$$\frac{\boxed{4d^2f^2a^2} + \boxed{4f^2\beta da^2} - 16\boxed{32}f^2d^2a^2 - 12\boxed{8}f^2\beta da^2}{\cancel{16a^2}} \qquad \frac{\boxed{-16df^2a^2\beta} - 3\boxed{4}f^2a^2\beta^2}{\cancel{16a^2}}$$

$\frac{\boxed{+4d^2f^2a^2}\boxed{+8f^2d\beta a^2}\boxed{+f^2\beta^2a^2}}{\cancel{16a^2}} \sqcap 0$. sive $16\cancel{f^2}d^2\cancel{a^2} - 12\cancel{f^2}\beta d\cancel{a^2} - 3\cancel{f^2}\cancel{a^2}\beta^2$ sive $16d^2 - 3\beta \frown 4d + \frac{9\beta^2}{4} \sqcap \frac{12\beta^2 + 9\beta^2}{4} \sqcap \frac{21}{4}\beta^2$. sive $\mp 4d \pm \frac{3\beta}{2} \sqcap \frac{\beta}{2}\sqrt{21}$. sive $d \sqcap \mp\frac{\beta\sqrt{21} + 3\beta}{8}$. Quodsi ergo nullus error in calculo, videtur haberi exitus.

Quod cum sit magni momenti repetemus calculum:

1 Quam *(1)* seriem *(2)* formulam *L*

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13 $g^2 \sqcap \frac{4d^2f^2 + 8f^2d\beta + f^2\beta^2}{16a^2}$: Der erste Term im Zähler müßte $16d^2f^2$ lauten; in Z. 14 müßten die beiden ersten Koeffizienten 16 lauten. Dieser Fehler beeinträchtigt die Rechnung bis Z. 16. Leibniz überprüft anschließend das Ergebnis durch eine korrekte Kontrollrechnung.

$$\begin{array}{cccc}
 & & \boxed{\begin{array}{l} \text{ǂ}\; cf\; y^2 \\ \\ \\ \text{‡}\; cf\; .. \end{array}} & \begin{array}{ll} \boxed{\text{ǂ}\;\; fda\; y} & \text{ǂ}\; f\beta da \\ \boxed{\begin{array}{l}\text{ǂ}\;\; f\beta c\; .. \\ \text{ǂ}\;\; gac\; ..\end{array}} & \boxed{\text{ǂ}\; ga^2d} \\ \boxed{\text{‡}\;\; fc\beta\; ..} & \text{‡}\; gac\beta \end{array} \\
 & cy\; + c\beta & & \begin{array}{ll} \boxed{\text{‡}\;\; fda\; ..} & \boxed{\text{‡}\; ga^2d} \end{array} \\
\text{ǂ}\; \dfrac{cy\; + da}{fy + ga} & \text{‡}\; \dfrac{\; + da}{\begin{array}{c} fy + f\beta \\ + ga \end{array}} & \sqcap & \dfrac{\boxed{\text{‡}\;\; gac\; ..}}{\begin{array}{lll} f^2y^2 & + 2fga\; y & + g^2a^2 \\ & + \;\; f^2\beta\; .. & + f\beta ga \end{array}}
\end{array}$$

sive $\dfrac{\text{ǂ} fda\beta\; \text{‡}\; gac\beta}{\begin{array}{l} f^2y^2 + 2fgay + g^2a^2 \\ \quad + \;\; f^2\beta\, .. + f\beta ga \end{array}}$ ut ante, diviso nominatore per f^2, fiet nominator,

$y^2 \dfrac{\begin{cases} + 2gay \\ + f\beta\;\; .. \end{cases}}{f} \quad \dfrac{\begin{cases} + g^2a^2 \\ + f\beta ga \end{cases}}{f^2}$. Unde faciendo $\dfrac{2ga + f\beta}{f} \sqcap \dfrac{1}{2}\beta$, sive $4ga + \textcircled{2} f\beta \sqcap \boxed{f\beta}$ fiet:

$g \sqcap \dfrac{-f\beta}{4a}$ et $g^2 \sqcap \dfrac{+f^2\beta^2}{16a^2}$. Iam $g^{\not 2}a^{\not 2} + f\beta \not g \not a \sqcap 0$ fiet $g \sqcap -\dfrac{f\beta}{a}$. quod contradicit priori, quare ut haberi possit exodus quemadmodum paulo ante feci, explicanda est y, et ponenda $y \sqcap z + d$.

Unde ex formula $y^2 \; \dfrac{\begin{cases} + 2fgay \\ + f^2\beta\; .. \end{cases}}{f^2} \; \dfrac{\begin{cases} + g^2a^2 \\ + f\beta ga \end{cases}}{f^2}$ ponendo $y \sqcap z + d$, et $y^2 \sqcap z^2 + 2dz + d^2$,

16 ex *(1)* aequatione *(2)* formula L

9 ut ante: S. 528 Z. 10; dort ist im Zähler $\beta = 1$ gesetzt. 13 contradicit priori: S. 529 Z. 13. 14 paulo ante: S. 529 Z. 3–11.

$$\text{fiet: } z^2 \begin{cases} + 2df^2 z \\ + 2fga.. \\ + f^2\beta\ .. \end{cases} \Big/ f^2 \quad \begin{cases} + d^2f^2 \\ + 2fgad \\ + f^2\beta d \\ + g^2a^2 \\ + f\beta ga \end{cases} \Big/ f^2 \quad \text{comparanda cum formula:}$$

$$z^2 \qquad + \frac{1}{2}\beta z \qquad\qquad * \quad .$$

Ergo $\dfrac{2df^{\not 2} + 2\not f ga + f^{\not 2}\beta}{f} \sqcap \dfrac{1}{2}\beta$. sive $4df + 4ga + \boxed{2}f\beta \sqcap \boxed{f\beta}$. Ergo $g \sqcap \dfrac{-4df - f\beta}{4a}$. et $g^2 \sqcap \dfrac{+16d^2f^2 + 8df^2\beta + f^2\beta^2}{16a^2}$. Quos valores inserendo in posteriore collatitia, fiet:

$16d^2f^2\not a^{\not 2} - 32f^2d^2\not a^{\not 2} - 8f^2\not a^{\not 2}d\beta + 16f^2\beta d\not a^{\not 2} + 16d^2f^2\not a^{\not 2} + 8df^2\beta\not a^{\not 2} + f^2\beta^2\not a^{\not 2}$
$-16f^2\beta d\not a^{\not 2} - 4f^2\beta^2\not a^{\not 2} \sqcap 0$,

sive $\left.\begin{array}{l} \boxed{\begin{array}{l} + 16d^2f^2 \\ \\ + 16d^2f^2 \\ - 32d^2f^2 \end{array}} \boxed{\begin{array}{l} - \ \ 8f^2d\beta \\ \\ + 16f^2\beta d \\ + \ \ 8df^2\beta \\ - 16f^2\beta d \end{array}} \begin{array}{l} + \ \ f^2\beta^2 \\ \\ - 4f^2\beta^2 \\ \\ \\ \end{array} \end{array}\right\} \sqcap 0$ quod est impossibile. Hac ergo methodo solvi problema non potest. Unde patet explicationem y per $z + d$. nihil contulisse.

Si z explicassemus per $y + h$, et z^2 per $y^2 + 2hy + h^2$, fieret:

$$\begin{array}{llll} y^2 & + & 2hy & + h^2 \\ & + & \frac{1}{2}\beta.. & + \frac{1}{2}\beta h \end{array}$$

$$\text{conferenda cum } y^2 \begin{cases} + 2fgay \\ + \ \ f^2\beta.. \end{cases} \Big/ f^2 \quad \begin{cases} + g^2a^2 \\ + f\beta ga \end{cases} \Big/ f^2 \quad .$$

Unde $g \sqcap \dfrac{2hf^{\not 2}\boxed{+\frac{1}{2}f^2\beta - f^2\beta} - \frac{1}{2}f^{\not 2}\beta}{2\not f a} \sqcap \dfrac{4hf - f\beta}{4a}$ et $g^2 \sqcap \dfrac{16h^2f^2 - 8hf^2\beta + f^2\beta^2}{16a^2}$. Et his valoribus in ultima aequatione collatitia insertis, fiet:

$$\boxed{f^2h^2} + \frac{1}{2}\beta hf^2 \sqcap \frac{\boxed{16h^2f^2} - 8hf^2\beta + f^2\beta^2}{16} + \frac{4f^2\beta h - f^2\beta^2}{4}$$

seu $\boxed{\begin{matrix} +\ 8\beta hf^2 \\ +\ 8hf^2\beta \\ -\ 16f^2\beta h \end{matrix}} \sqcap f^2\beta^2 - 4f^2\beta^2 \sqcap 0$. quod est absurdum.

Scilicet generaliter: ipsi $y^2 \ \frac{\begin{cases} 2fgay \\ f^2\beta\ .. \end{cases}}{f^2} \ \frac{\begin{cases} +\,g^2a^2 \\ +f\beta ga \end{cases}}{f^2}$ aequentur $y^2 + ly + ma$. et sumendo l, et m, quasi cognitas, fiet:

$g \sqcap \dfrac{fl - f\beta}{2a}$. et $g^2 \sqcap \dfrac{f^2l^2 - 2f^2l\beta + f^2\beta^2}{4a^2}$, quo valore in posteriore collatitia inserto, fiet: $\cancel{f^2}l^2\boxed{-2f^2l\beta}\boxed{+f^2\beta^2}, \boxed{+2f^2\beta l} - \textcircled{2}\cancel{f^2}\beta^2 \sqcap \cancel{f^2}ma$. et deletis caeteris arbitrariis, fit aequatio: $l^2 - \beta^2 \sqcap ma$. Unde fit ut quandocunque ma. abest, fiat $l \sqcap \beta$. id est fiat progressio triangularis. Posita autem ma, adesse, series haberi poterit: $\dfrac{1}{y^2 + ly + l^2 - \beta^2}$ sumtis l. et β. pro arbitrio sed constantibus, et β. differentia abscissarum. Quodsi l. et β. aequales oritur series triangularis, si inaequales, v. g. ponendo $l \sqcap 2$. et $\beta \sqcap 1$. fiet:

$$\frac{1}{1+2+4-1} \quad \frac{1}{4+4+4-1} \quad \frac{1}{9+6+4-1} \quad \frac{1}{16+8+4-1} \quad \frac{1}{25+10+4-1} \quad \frac{1}{36+12+4-1} \quad \text{etc.}$$
$$\frac{1}{6} \quad \frac{1}{11} \quad \frac{1}{18} \quad \frac{1}{27} \quad \frac{1}{38} \quad \frac{1}{51}$$

Si potius malis $l \sqcap 1$. et $\beta \sqcap 2$. tunc $\dfrac{1}{y^2 + ly + l^2 - \beta^2}$ dabit:

$$\frac{1}{1+1-3} \quad \frac{1}{4+2-3} \quad \frac{1}{9+3-3} \quad \frac{1}{16+4-3} \quad \frac{1}{25+5-3}$$
$$\frac{1}{-1} \quad \frac{1}{3} \quad \frac{1}{9} \quad \frac{1}{17} \quad \frac{1}{27} \quad \text{etc.}$$

Generaliter fractionum iniri poterit summa, si a nominatoribus triangularium fractionum constans quaedam quantitas subtracta intelligatur. Non perinde si addita.

2–5 absurdum. *(1)* Scilicet 2fga + $f^2\beta$, et g^2a^2 + fβga *(2)* Scilicet *L* 12 abscissarum. *(1)* | Itaque seriei huiusmodi: *streicht Hrsg.* | $\dfrac{1}{1+1+1-1} + \dfrac{1}{4+2+1-1} +$ *(2)* Quodsi *L* 19 si *(1)* ad nominatores triangularium fractionum constans quaedam quantitas addita vel ab iis *(2)* a nominatoribus *L*

Generalius omnium fractionum iniri potest summa, quarum numerator unitas, nominatores vero, quadrati numerorum arithmetice crescentium, aucti ipsis radicibus suis per datam multiplicatis et praeterea excessu quadrati datae, super quadratum intervalli ipsius arithmeticae progressionis. Qui excessus nihilo minor esse potest, cum data minor est intervallo; et nihilo aequalis est, cum data et intervallum aequales, quo postremo casu res redit ad seriem fractionum triangularium.

Sed ut videamus an non aequatio: $y^2+\frac{1}{2}\beta y$ altiorem seriem, v. g. ad y^4 assurgentem habeat summatricem, ideo multiplicando per $\frac{e^2}{a^2}y^2+hy+la$ fiet:

$$\begin{array}{lllllll} \frac{e^2}{a^2}y^4 & + & \frac{1}{2}\beta\frac{e^2}{a^2}y^3 & + & \frac{1}{2}h\beta y^2 & + & \frac{1}{2}la\beta y \quad * \\ & + & \quad h \quad .. & + & \quad la \;\; .. & & \end{array}$$

conferenda cum facto ex $\frac{e}{a}y^2+fy+ga$ in seipsum, et praeterea in $\begin{array}{l}\frac{2e}{a}\beta y+\frac{e}{a}\beta^2\\ \qquad\qquad + f\beta\end{array}$

1–6 *Am Rande, zweite und dritte Spalte gestrichen:*

$$\begin{array}{ccc} 1 & 3 & \\ & & 3 \\ 2 & 6 & \\ & & 3 \\ 3 & 9 & \\ & & 3 \\ 4 & 12 & \end{array}$$

2 crescentium, *(1)* ipsis radicibus suis per datam multiplicati, *(2)* aucti *L* 3 praeterea *(1)* differentia inter huius datae, et *(2)* excessu *(3)* excess *(4)* residuo quadrati d *(5)* excessu *L* 4 potest, cum *(1)* quadratum datae *(2)* data *L*

fiet:

$$\begin{array}{llllllllll}
\frac{e^2}{a^2}y^4 & + & \frac{2e}{a}f\ y^3 & + & \frac{2e}{a}\ ga\ y^2 & + & 2\ fga\ y & + & g^2a^2 \\
 & & & + & f^2\ .. & & & & \\
 & + & \frac{2e^2}{a^2}\beta\ .. & + & \frac{2e}{a}\ f\beta\ .. & + & \frac{2e}{a}\ \beta ga\ .. & & \\
 & & & + & \frac{e^2}{a^2}\ \beta^2\ .. & + & \frac{e}{a}\ f\beta^2\ .. & + & \frac{e\beta^2}{a}ga \\
 & & & + & \frac{e}{a}\ \beta f\ .. & + & f^2\ \ \beta\ \ .. & + & f\beta ga
\end{array}$$

Conferendo terminos ultimos fiet: $g^{\not 2}a^2 + e\beta^2\not g + f\beta\not g a \sqcap 0.$ et $g \sqcap \frac{-e\beta - fa, \frown\ \beta}{a^2}.$

Conferendo penultimos, fiet, $l \sqcap \frac{2fga + 2e\beta g + \frac{e}{a}f\beta^2 + f^2\beta}{\frac{1}{2}a\beta}$ quo valore ipsius l, in antepenultimorum aequatione inserto, fiet:

$$h \sqcap \frac{\frac{2e}{\not a}g\not a + f^2 + \frac{2e}{a}f\beta + \frac{e^2}{a^2}\beta^2 + \frac{e}{a}\beta f - \frac{2fga + 2e\beta g + \frac{e}{a}f\beta^2 + f^2\beta}{\frac{1}{2}\beta}}{\frac{1}{2}\beta}.$$

Unde novissima aequatio haec erit ex terminis secundis collatis:

$$\frac{1}{2}\beta\frac{e^2}{a^2} + \frac{2eg + f^2 + \frac{2e}{a}f\beta + \frac{e^2}{a^2}\beta^2 + \frac{e}{a}f\beta - \frac{2fga + 2e\beta g + \frac{e}{a}f\beta^2 + f^2\beta}{\frac{1}{2}\beta}}{\frac{1}{2}\beta} \sqcap + \frac{2ef}{a} + \frac{2e^2}{a^2}\beta.$$

et explicando g. multiplicandoque omnia per $a^2\beta^2$, et omnino fractiones tollendo fiet:

$$\underset{\text{IIII}}{\boxed{\beta^3e^2}} - 7\underset{\text{IIII}}{\boxed{8}}\beta^3e^2\underset{\text{V}}{\boxed{+4f^2a^2\beta}}\underset{\text{III}}{\boxed{+8efa\beta^2}}\underset{\text{I}}{\boxed{+4e^2\beta^3}}\ \underset{\text{II}}{\boxed{+4ef\beta^2a}} - 16fga^3 - 16e\beta ga^2$$

$$\underset{\text{III}}{\boxed{-8ef\beta^2a}} - 4\underset{\text{V}}{\boxed{8}}f^2\beta a^2 \sqcap \underset{\text{II}}{\boxed{4efa\beta^2}}\underset{\text{I}}{\boxed{+4e^2\beta^3}}$$ et fiet denique:

$+7\beta^3e^2 + 16fga^3 + 16e\beta ga^2 + 4f^2\beta a^2 \sqcap 0.$ et explicando ipsam g. fiet:

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12 fiet: Leibniz vergißt auf der linken Seite der folgenden Gleichung den Term $-8efa\beta^2$, was das Ergebnis der Umformung beeinträchtigt. Die folgende Kontrollrechnung ist, wie Leibniz selbst bemerkt, fehlerhaft.

$$\underset{II}{\boxed{7\beta^3e^2}} - 32\underset{III}{\boxed{16}}fae\beta^{\not 2} - 12\underset{I}{\boxed{16}}f^2a^2\not\beta - 9\underset{II}{\boxed{16}}e^2\beta^{\not 3 2}\underset{III}{\boxed{-16e\beta^2fa}}\underset{I}{\boxed{+4f^2\beta a^2}} \sqcap 0.$$ sive $+32fae\beta + 12f^2a^2 + 9e^2\beta^2 \sqcap 0.$

Operae pretium est eundem calculum bis facere ob eius momentum quae melior probandi ratio est, et sane expeditissima.

$y^2 + \frac{1}{2}\beta y$, multiplicetur per $\frac{e^2}{a^2}y^2 + hy + la$, fiet:

$$\begin{array}{llllllll} \frac{e^2}{a^2}y^4 & + & \frac{1}{2}\beta\frac{e^2}{a^2}y^3 & + & \frac{1}{2}\beta hy^2 & + & \frac{1}{2}\beta lay & * \\ & + & h \;.. & + & la \;.. & & & \end{array}$$

conferenda cum formula sequente:

$$\begin{array}{lllllllll} \frac{e^2}{a^2}y^4 & + & \frac{2e}{a}f \;\; y^3 & + & 2eg \;\; y^2 & + & 2fga \;\; y & + & g^2a^2 \qquad \odot \\ & + & \frac{2e^2}{a^2}\beta \;.. & + & f^2 \;.. & + & 2e\beta g \;.. & + & e\beta^2 g \\ & & & + & \frac{3\boxed{2}e}{a}f\beta \;.. & + & \frac{ef\beta^2}{a} \;.. & + & f\beta ga \\ & & & + & \frac{e^2}{a^2}\beta^2 \;.. & + & f^2\beta \;.. & & \\ & & & \boxed{+} & \boxed{\frac{e}{a}\beta f \;..} & & & & \end{array}$$

Has duas formulas inter se conferendo, quia ultima posterioris nihilo seu loco vacuo prioris aequalis est, fiet:

$g \sqcap \frac{-e\beta^2 - f\beta a}{a^2}$ et conferendo penultimos terminos, fiet:

$l \sqcap \frac{4fga^2 + 4e\beta ga + 2ef\beta^2 + 2f^2\beta a}{\beta a^2}$ et conferendo antepenultimos, fiet:

$$h \sqcap \frac{\boxed{4ega^2\beta}\boxed{+2f^2a^2\beta} + 6eaf\beta^2 + 2e^2\beta^3 - 8fga^3 - 4\boxed{8}e\beta ga^2 - 4ef\beta a^2 - 2\boxed{4}f^2\beta a^2}{\beta^2a^2}$$

et conferendo secundos, fiet:

18 *Über* $-4ef\beta a^2$: Hinc ob positam βa^2, pro $\beta^2 a$, error in sequentibus.

$\beta^2 e^2 \underset{IIII}{\boxed{+\,8ega^2\beta}}\underset{III}{\boxed{+\,4f^2a^2\beta}}\underset{II}{\boxed{+\,12eaf\beta^2}}\underset{I}{\boxed{+\,4e^2\beta^3}} - 16fga^3 - 8\underset{IIII}{\boxed{16}}e\beta g a^2 \underset{II}{\boxed{-\,8ef\beta a^2}} -$ $4\boxed{8}f^2\beta a^2 \sqcap \underset{II}{\boxed{4efa\beta^2}}\underset{I}{\boxed{+\,4e^2\beta^3}}$, sive,

$\beta^3 e^2 - 16fga^3 - 8e\beta g a^2 - 4f^2\beta a^2 \sqcap 0$. et explicando g, fiet:

$\beta^3 e^2 + 16fae\beta^2 + 12\underset{I}{\boxed{16}}f^2\beta a^2 + 8e^2\beta^3 + 8e\beta^2 af\boxed{-4f^2\beta a^2}$ sive

$9\beta^{\not{3}2}e^2 + 32fae\beta^{\not{2}} + 12f^2\not{\beta}a^2 \sqcap 0$. prorsus ut supra, ut adeo de hoc calculo possimus esse certi.

Unde $\beta^2e^2 + \dfrac{32}{9}fae\beta + \dfrac{32 \frown 32}{81, \frown 4}f^2a^2 \sqcap \dfrac{32,32}{81,4}f^2a^2 - \dfrac{12f^2a^2}{9}$.

Unde $\mp\beta e \mp \dfrac{32fa}{18} \sqcap fa\sqrt{\dfrac{32,32,, - 12,9,4}{81,4}} \sqcap \dfrac{fa\sqrt{592}}{18}$. sive $e \sqcap \dfrac{fa}{18\beta} \frown -32 \mp \sqrt{592}$, sive $e \sqcap \dfrac{fa}{\beta} \frown \dfrac{-32 \mp \sqrt{592}}{18}$. Unde patet e esse quantitatem negativam. Hic valor $e \sqcap \dfrac{-32fa \mp fa\sqrt{592}}{18\beta}$, inseratur valori g, erit $g \sqcap \dfrac{+32fa\beta^2 \pm fa\beta^2\sqrt{592} - f\beta^2 a}{\beta a^2}$, sive $g \sqcap \dfrac{31f\beta \pm f\beta\sqrt{592}}{a}$.

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6 *Neben* certi: Imo inest error.

7–11 *Nebenrechnungen und Nebenbetrachtungen:*

$$\begin{array}{rrrr} 32 & 81 & 36 & 1024 \\ 32 & 4 & 12 & 432 \\ \hline 64 & 324 & 72 & 592 \\ 96 & & 36 & \\ \hline 1024 & & 432 & \end{array}$$

10–537,1 g $\sqcap \dfrac{31f\beta \pm f\beta\sqrt{592}}{a}$. *(1)* Ita *(2)* Sed quoniam tota fractio *(3)* Multiplicando *L*

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5 ut supra: S. 535 Z. 2. 10 erit g: Konsequent wäre $g \sqcap \dfrac{32fa\beta^2 \pm fa\beta^2\sqrt{592} - 18f\beta^2 a}{18\beta a^2} \sqcap \dfrac{14f\beta \pm f\beta\sqrt{592}}{18a}$. 13 error in sequentibus: Der Fehler beeinträchtigt die Rechnung bis Z. 11 sowie in N. 38₁₇.

Multiplicando autem per $\frac{e^2}{a^2}y^2 + hy + la$ fractionis $\frac{1}{y^2 + \frac{1}{2}\beta y}$ non tantum nominatorem, ut iam praestitimus, sed et numeratorem fiet:

$$\frac{1}{y^2+\frac{1}{2}\beta y} \sqcap \frac{\frac{e^2}{a^2}y^2 + hy + la \qquad ☽}{\frac{e^2}{a^2}y^4 + \frac{1}{2}\beta\frac{e^2}{a^2}y^3 + \frac{1}{2}\beta hy^2 + \frac{1}{2}\beta lay \quad * \quad ☽ \atop + \quad h \;.. \;+\; la\; ..},$$

cuius nominatorem cum nominatori fractionis generalis summam seriei recipientis contulerimus cum successu, unum superest, ut si licet eadem felicitate et numeratores conferamus. Quod antequam faciam, repetendus est calculus, quo fractionis generalis numerator habitus est.

M a l o   e n i m   b i s   i d e m   a g e r e ,   q u a m   s e m e l   n i h i l.

Setzung: ǂ $\sqcap \frac{p}{a}$. $\sqrt{592} \sqcap \frac{q}{a}$.

Berechnung der Größen e, g, l, h auf LH 35 XII 2 Bl. 62 r°:

$e \sqcap \frac{-32fa^2 + fpq}{18\beta[a]}$ $\qquad$ $g \sqcap \frac{31f\beta[a^2] - [f]p\beta q}{[a^3]}$

$l \sqcap \frac{4fga^2 + 4e\beta ga + 2ef\beta^2 + 2f^2\beta a}{\beta a^2}$

$h \sqcap \frac{6eaf\beta^2 + 2e^2\beta^3 - 8fga^3 - 4e\beta ga^2}{}$ [*bricht ab*]

12 a *erg. Hrsg.* 12 $\frac{31f\beta a - p\beta q}{a}$ *L ändert Hrsg.* 14 *(1)* h $\sqcap$ $\frac{-4ega^2\beta - 2f^2a^2\beta + 6eaf\beta^2 + 2e^2\beta^3 - 8fga^3}{}$ *(2)* h $\sqcap$ ... *L*

2 praestitimus: S. 535 Z. 6 f.

$$\text{Fractio composita erat: } \text{ǂ}\,\frac{\frac{b}{a}y^2+cy+da}{\frac{e}{a}y^2+fy+ga}\;\text{‡}\;\frac{\begin{array}{l}\frac{b}{a}\;\;y^2+cy+da\\ \frac{2b}{a}\beta y\;+c\beta\\ \;\frac{b}{a}\beta^2\end{array}}{\begin{array}{l}\frac{e}{a}\;\;y^2+fy+ga\\ \frac{2e}{a}\beta y\;+f\beta\\ \frac{e}{a}\beta^2\end{array}}$$

Unde multiplicando nominatorem per nominatorem habuimus nominatorem illum signo ⊙. cum altero signo ☽. hactenus collatum: restat ut numeratorem multiplicatione per crucem coniunctis productis factum, sub finem plagulae praecedentis positum, et sub initium sequentis repetito calculo examinandum cum numeratore signi ☽. conferamus.

Vid. plag. seq.

1–5 *Daneben isolierte Rechnung, schräg geschrieben:*

30
2
—
60
8
—
480

---

3 erat: N. 38_{15} S. 524 Z. 12–16. 7 signo ⊙: S. 535 Z. 9–13. 8 praecedentis: N. 38_{15} S. 525 Z. 1–16. 9 sequentis: N. 38_{17} S. 539 Z. 7 – S. 540 Z. 16.

38$_{17}$. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS QUINTA DECIMA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 32–33. 1 Bog 2°. Ca 2 1/2 S. Bl. 33 v° leer. Cc 2, Nr. 775 A tlw.

Pars XV

Nimirum fiet ordinando:

$$\begin{array}{lll}
\text{formula multiplicanda} & & \dfrac{e}{a}\,y^2 + \dfrac{2e}{a}\,\beta y + \dfrac{e}{a}\,\beta^2 \\
 & & \quad + \quad f\,..\, + f\beta \\
 & & \qquad\qquad + ga \\
\text{per formulam} & ⸸, & \left\{ \dfrac{b}{a}\,y^2 + \quad cy + da \right.
\end{array}$$

$$⸸\left\{\begin{array}{llllll}
 & & & \boxed{edy^2}\,[/////] & 2e\beta dy & e\beta^2 d \\
 & & & & \boxed{fda\,..}\,[//] & f\beta da \\
 & & & & & \boxed{ga^2 d} \\
 & & \boxed{\dfrac{e}{a}\,cy^3}\,[/] & \dfrac{\boxed{2}\,e}{a}\,[////]\,\beta c\,.. & \dfrac{e}{a}\beta^2 c\,.. & \\
 & & & \boxed{fc\,..}\,[//] & \boxed{f\beta c\,..}\,[/] & \\
 & & & & \boxed{gac\,..} & \\
\boxed{\dfrac{eb}{a^2}\,y^4} & \boxed{\dfrac{2e\beta b}{a^2}\,..}\,[//] & \boxed{\dfrac{e\beta^2 b}{a^2}\,..}\,[///] & & & \\
 & \boxed{\dfrac{fb}{a}\,..}\,[///] & \boxed{\dfrac{f\beta b}{a}\,..}\,[/] & & & \\
 & & \boxed{gb\,..} & & &
\end{array}\right.$$

Hoc productum concordat cum superiore sub finem plagulae praecedentem praecedentis, itaque dubitandum non est de calculi veritate.

Eodem modo fiet ordinando,

$$\text{formula multiplicanda } \left.\begin{array}{lll} \frac{b}{a}\,y^2 & \frac{2b\beta}{a}\,y & \frac{b}{a}\,\beta^2 \\ & c\,.. & c\beta \\ & & da \end{array}\right\}\ddagger$$

$$\text{per formulam: } \frac{e}{a}\,y^2 + \quad f\,.. + \; ga$$

$$\left.\begin{array}{ccccc}
 & & \boxed{bgy^2} & 2b\beta gy & b\beta^2 g \\
 & & & \boxed{cga\,..} & c\beta ga \\
 & & & & \boxed{dga^2} \\
 & \underset{///}{\boxed{\frac{b}{a}\,fy^3}} & \frac{\underset{/}{\textcircled{2}}\,b\beta f}{a}\,.. & \frac{b}{a}\,\beta^2 f\,.. & \\
 & & \underset{//}{\boxed{cf\,..}} & \underset{/}{\boxed{c\beta f\,..}} & \\
 & & & \underset{//}{\boxed{daf\,..}} & \\
\boxed{\frac{be}{a^2}\,y^4} & \underset{//}{\boxed{\frac{2b\beta e}{a^2}\,..}} & \underset{///}{\boxed{\frac{b\beta^2 e}{a^2}\,..}} & & \\
 & \underset{/}{\boxed{\frac{ce}{a}\,..}} & \underset{////}{\boxed{\frac{c\beta e}{a}\,..}} & & \\
 & & \underset{/////}{\boxed{\frac{d\not{a}e}{\not{a}}\,..}} & &
\end{array}\right\}\ddagger$$

Hoc productum rursus convenit cum superiore quod est sub finem plagulae praecedentem praecedentis possumusque de hac quoque calculi parte securi esse.

1 f. praecedentem praecedentis: N. 38_{15} S. 525 Z. 1–7 17 f. praecedentem praecedentis: N. 38_{15} S. 525 Z. 8–16

Et quoniam haec duo producta contrariis signis afficiuntur, destruantur quae conveniunt.

A b s o l u t i s e r g o d e s t r u c t i o n i b u s , r e s t a b i t n u m e r a t o r t o t a l i s ,

$$\text{nempe}\left\{\begin{array}{llll} \text{ǂ}\frac{e}{a}\beta c y^2 & \text{ǂ}\ 2e\beta d y & \text{ǂ}\ e\beta^2 d & \\ \ddagger\frac{b}{a}\beta f.. & \text{ǂ}\frac{e}{a}\beta^2 c\,.. & \text{ǂ} f\beta d a & \\ & \ddagger\ 2b\beta g\,.. & \ddagger\ b\beta^2 g & \\ & \ddagger\frac{b}{a}\beta^2 f\,.. & \ddagger\ c\beta g a & \end{array}\right.$$

in quo literae, a. β. cognitae: literae autem e. et g, h. pendent a f. ac proinde explicanda, si f. quaerimus. Denique, $b.\,c.\,d$. sunt indeterminatae a se invicem independentes: quas tentemus determinare, conferendo cum superiore numeratore, seu formula:

$$e^2y^2 \qquad + ha^2y \qquad + la^3$$

ubi h. et l. quoque pendent ab f. Tamdiu ergo differemus quaerere f. quamdiu non cogemur.

Ex prima collatione fiet $b \sqcap \ddagger\dfrac{e^2a + e\beta c}{\beta f}$.

Ex secunda: $\text{ǂ}2e\beta d a f\ \text{ǂ}\ e\beta^2 c f + 2e^2a^2g \ddagger 2e\beta a c g + 2e^2a\beta f \ddagger 2e\beta^2 c f \sqcap h a^3 f$ et fiet $c \sqcap \dfrac{ha^3f \ddagger 2e\beta d a f - 2e^2a^2g - 2e^2a\beta f}{\text{ǂ}e\beta^2 f \ddagger 2e\beta a g \ddagger 2e\beta^2 f}$.

Ex tertia: $\text{ǂ}e\beta^2 d\ \text{ǂ}\ f\beta d a \ddagger b\beta^2 g \ddagger c\beta g a \sqcap la^3$, sive $\text{ǂ}e\beta^2 d\ \text{ǂ}\ f\beta d a + \dfrac{e^2a\beta^2 g \ddagger e\beta^3 c g}{\beta f} \ddagger c\beta g a \sqcap la^3$.

5–8 *Neben dem Schema*: $\frac{p}{a} \sqcap \text{ǂ}$.

13 f. Tamdiu ... cogemur. *erg. L*

---

20 $\frac{p}{a} \sqcap \text{ǂ}$: vgl. N. 3816 S. 537 Z. 10.

Quoniam vero quaerimus d, et in $c.$ latet d, ideoque $c.$ per brachylogiam ita exprimemus: $c \sqcap \dfrac{ma^4 ‡ 2e\beta daf}{na^3}$, adeoque

$ǂ e\beta^2 dna^3\beta f ǂ f\beta dana^3\beta f + e^2a\beta^2gna^3 ‡ e\beta^3ma^4g + 2e\beta^3e\beta dafg ‡ ma^4\beta ga\beta f + 2e\beta daf\beta ga\beta f \sqcap la^3na^3\beta f$ eritque

$$d \sqcap \frac{la^3na^3\beta f - e^2a\beta^2gna^3 ǂ e\beta^3ma^4g ǂ ma^4\beta ga\beta f}{ǂ e\beta^2na^3\beta f ǂ f\beta ana^3\beta f + 2e\beta^3e\beta afg + 2e\beta af\beta ga\beta f}.$$

Posito ergo $f.$ esse quamlibet cognitam pro arbitrio sumtam, etiam $e. g. h. m. n.$ cognitae erunt, et satisfactum est collationi.

Quoniam patet superesse f supernumerariam, nec tamen ab ea possit oriri variatio, cum problema sit determinatum, ideoque necesse est eam destrui. Quod ut videamus, utile est quamlibet literam quae f continet, explicari; eatenus saltem, quatenus f continet, itaque pro g ponemus $\dfrac{\gamma f}{a}$, pro e ponemus: $\dfrac{Ef}{a}$, pro h ponemus $\dfrac{Hf^2}{a}$, pro l ponemus $\dfrac{\lambda f^2}{a^2}$, pro m ponemus $\dfrac{\mu f^3}{a^3}$, et pro $n \sqcap \dfrac{Nf^2}{a^2}$, sed hoc credo sub exitum sufficere, ubi destruendi f tempus erit.

Antequam autem explicemus $d,\ c,\ b$, explicabimus distincte l, et h:

$$\beta a^2 l \sqcap 4fa^{\cancel{2}}, + 4\beta\cancel{a} \frown \llcorner \frac{-32fa ǂ fa\sqrt{592}}{18\beta} \lrcorner,, \frown \frac{31f\beta ‡ f\beta\sqrt{592}}{\cancel{a}},,,$$

$$\frac{-64f^2a\beta^2 ǂ 2f^2a\beta^2\sqrt{592}}{18\beta} + 2f^2\beta a$$

$$\text{sive } l \sqcap \frac{\begin{matrix} 18\beta 4fa31f\beta ‡ 18\beta 4faf\beta\sqrt{592} - 4\beta 31fa31f\beta ǂ 4\beta 32faf\beta\sqrt{592} \\ ǂ 4\beta fa\sqrt{592}, 31f\beta - 4\beta fa\sqrt{592}, f\beta\sqrt{592} - 64f^2a\beta^2 ǂ 2f^2a\beta^2\sqrt{592} + 18\beta 2f^2\beta a \end{matrix}}{18\beta^2a^2}$$

14 l, et h *(1)*, est autem *(a)* l $\sqcap$ 128f^{2}aβ *(b)* l $\sqcap$ 124fβa ‡ 4fβa$\sqrt{592}$ *(2)*: *(a)* Sed ut hoc quoque tutius procedat, utile arbitror, numeros adhibere, quo facilius error vitetur *(b)* βa^{2}l *L*

---

14 l, et h: Leibniz verwendet für g den falschen Wert aus N. 38₁₆ S. 536 Z. 11; konsequent gerechnet, müßte in Z. 17 der dritte Term des Zählers $-4\beta 32fa31f\beta$ lauten. Leibniz vermutet schließlich einen Fehler und beginnt eine Kontrollrechnung.

Unde $l \sqcap \dfrac{-4008f^2 \ddagger 182f^2\sqrt{592}}{18a}$.

Cum vero etiamnum verear, ne aliquis in ipsa denominatorum collatione error insit, resumere placet tertia vice, et quidem adhibitis numeris, necesse est enim, ut me assuefaciam usui numerorum in calculo analytico. Esto $a \sqcap 1$. $\beta \sqcap 4$. $f \sqcap 2$. $e \sqcap 3$. et fiet $\dfrac{-2f4\beta a - 3e16\beta^2 + 57\lambda a}{[a^2]} \sqcap g$.

$$70l \sqcap \frac{\overset{\not 8}{\boxed{4}}2fga^2 + \overset{\not 3}{\boxed{4}}3e4\beta ga + \overset{\not 3}{\boxed{2}}3e2f16\beta^2 + \overset{\not 5}{\boxed{2}}4f^2 4\beta a}{4\beta a^2}$$

$$78h \sqcap \frac{\overset{\not 3}{\boxed{4}}3ega^2 + \overset{\not 8}{\boxed{2}}4f^2a^2 + \overset{\not 9}{\boxed{6}}3e2f4\beta a + \overset{\not 9}{\boxed{2}}9e^2 16\beta^2 - \overset{\not 4}{\boxed{2}}70la^3}{4\beta a^2}$$

---

1 *Nebenrechnungen:*

18	−592	−31	$\not 2$	‡ 32	‡2	‡18	‡31
4	4	31	$\not 2$ $\not 4$ 4	4		4	4
72	−2368	31	$\not 4$ $\not 0$ $\not 0$ 8 $\not f$ 2 2 2	‡128		‡72	‡124
31		93	$\not 1$ $\not 8$ $\not 8$ 8				‡128
72		961	$\not 1$ 1				‡ 2
216		4					$‡254\sqrt{592}$
+ 2232		−3844					$‡\ 72\sqrt{592}$
+ 36		−2368					$‡182\sqrt{592}$
+ 2268		− 64					
− 6276		−6276					
− 4008							

5 fiet *(1)* $g \sqcap \dfrac{-2f4\beta a - 3e16\beta^2}{a}$ *(2)* | $\dfrac{-2f4\beta a - 3e16\beta^2 + 57\lambda a}{a}$ *ändert Hrsg.* | $\sqcap g$ *L*

---

5 $+57\lambda a$: Leibniz verwendet für g zunächst den Wert der Stufe *(1)* der Lesart, dann addiert er im Zähler $57\lambda a$, um für die Kontrollzahl von g den Wert $1 = \dfrac{-56+57}{1}$ zu erhalten.

et pro 70 l substituendo eius valorem inventum multiplicandoque omnia per $4\beta a$, ut et communem hunc divisorem habere possint fiet

$$\overset{\text{VI}}{78h} \sqcap \frac{\boxed{\overset{\text{III}}{\boxed{4}\,3ega^2 4\beta}} + \boxed{\overset{\text{V}}{\boxed{2}\,4f^2a^2 4\beta}} + \boxed{2}\boxed{6}\overset{\text{IX}}{3e2f16\beta^2 a} + \boxed{2}\overset{\text{IX}}{9e^2 64\beta^3} - \boxed{8}\overset{\text{II}}{2fga^3} - \boxed{4}\boxed{8}\overset{\text{III}}{3e4\beta ga^2}\ \boxed{-\boxed{4}\overset{\text{III}}{3e2f16\beta^2 a}} - \boxed{2}\boxed{4}\overset{\text{VIII}}{4f^2 4\beta a^2}}{\overset{\text{VII}}{16\beta^2 a^2}}$$

sive $\overset{\cancel{6}}{78h} \sqcap \dfrac{\overset{\cancel{3}}{\boxed{2}}3e2f16\beta^2 a + \overset{\cancel{9}}{\boxed{2}}9e^2 64\beta^3 - \overset{\cancel{2}}{\boxed{8}}2fga^3 - \overset{\cancel{6}}{\boxed{4}}3e4\beta ga^2 - \overset{\cancel{4}}{\boxed{2}}4f^2 4\beta a^2}{16\beta^2 a^2}$ et conferendo terminos secundos, fiet:

$\boxed{4\beta 9e^2} + \boxed{2}\,78ha^2 \sqcap \boxed{4}\,3ea2f + \boxed{3}\boxed{4}\,9e^2 4\beta + 24\,\mu a^2$ sive explicata h, fiet:

$\boxed{\boxed{4}\,3e2f16\beta^2 a} + \boxed{4}\,9e^2 64\beta^3 - \overset{\cancel{4}}{\boxed{16}}\,2fga^3 - \overset{\cancel{3}}{\boxed{8}}\,3e4\beta ga^2 - \overset{\cancel{8}}{\boxed{4}}\,4f^2 4\beta a^2 \sqcap$

$\boxed{\boxed{4}\,16\beta^2 3ea2f} + \boxed{\boxed{3}\,16\beta^2 9e^2 4\beta} + \overset{\cancel{6}}{16\beta^2 24}\,\mu a^2$.

Dividi autem possunt omnia per β. Porro ordinando ut radix extrahi possit seu haberi e absolute; fiet:

2–4 fiet *(1)* | $78\text{h} \sqcap \dfrac{\overset{\cancel{\text{III}}}{\boxed{4}}3\text{ega}^2 4\beta\ \boxed{+\overset{\cancel{\text{V}}}{\boxed{2}}4\text{f}^2\text{a}^2 4\beta} + \boxed{4}\boxed{6}\overset{\cancel{\text{IX}}}{3\text{e}2\text{f}16\beta^2\text{a}} + \overset{\cancel{\text{IX}}}{\boxed{2}9\text{e}^2 64\beta^3} - \overset{\cancel{\text{II}}}{\boxed{8}}2\text{fga}^3 - \boxed{4}\boxed{8}\overset{\cancel{\text{III}}}{3\text{e}^4\beta\text{ga}^2} - \boxed{\overset{\text{III}}{\boxed{4}3\text{e}2\text{f}16\beta^2\text{a}}} - \boxed{4}\overset{\cancel{\text{VIII}}}{4\text{f}^2 4\beta\text{a}^2}}{16\beta^2\text{a}^2}$. *(!) streicht Hrsg.* | Porro $\dfrac{\text{e}^2}{\text{a}^2}$ *(2)* $\overset{\text{VI}}{78\text{ h}}$ *L*   8 $-\overset{\cancel{4}}{\boxed{16}}2\text{fga}^3$ *erg. L*

---

4 $\overset{\text{VI}}{78h} \sqcap$: Leibniz führt die Neunerprobe vor dem Reduzieren der Terme im Zähler durch.

8 $-\overset{\cancel{4}}{\boxed{16}}\,2fga^3$: Leibniz hatte den Term zunächst vergessen. Nach dem Einfügen korrigiert er die durch weitere Flüchtigkeitsfehler beeinträchtigte Rechnung nicht mehr, sondern setzt auf dem nächsten Bogen neu an. Die Teilbarkeitsbemerkung hinsichtlich β trifft nach der Ergänzung nicht mehr zu.

$$9e^2 - \boxed{8}\,\frac{ga^2}{16\beta^2}3e + \frac{g^2a^4}{\boxed{4}\,256\beta^4} \sqcap \frac{g^2a^4 + \boxed{16}\,4f^2a^2 16\beta^2 + \boxed{4}\,4\beta 24\,\mu a^2 16\beta^2}{\boxed{4}\,256\beta^4}$$

$$\text{sive } \mp 3e \pm \frac{ga^2}{\boxed{2}\,16\beta^2} \sqcap \frac{\sqrt{g^2a^4 + \boxed{16}\,\beta^2 4f^2a^2 + \boxed{4}\,4\beta 24\,\mu a^2 16\beta^2}}{\boxed{2}\,16\beta^2}$$

---

2 *Nebenrechnungen:*

24	*Dazu gestrichen* :	16
16	1 3	16
144	~~6~~ ~~2~~ 0 9	256
24	7 7	4
384	~~4~~ ~~9~~ 4 9 [*!*]	1024
4	1	6144
1536		1
4	*Zur ersetzten Lesart,*	7169
6144	*nicht gestrichen* :	
64	1 2	7
1	~~6~~ ~~1~~ 6 1	~~7~~ ~~1~~ 6 9
6209	7 9	8
	~~4~~ ~~9~~ 4 9	~~6~~ ~~4~~ 6
	1	1
		[*Rechnung bricht ab*]

12–15 *linke Spalte* *(1)* 6144 16 1 6161 *(2)* 6144 64 1 6209 *L*

38_{18}. DE SERIERUM SUMMIS ET DE QUADRATURIS PARS SEXTA DECIMA

Überlieferung: *L* Konzept: LH 35 V 4 Bl. 34–36. 1 1/2 Bog. 2°. Ca 3 1/2 S. Bl. 35 v° u. 36 v° leer. Textfolge Bl. 34 r°, 34 v°, 36 r°, 35 r°.
Cc 2, Nr. 775 B, C.

[*1. Ansatz*]

(1) $g \sqcap \dfrac{-e\beta^2 - f\beta a}{a^2}$ ex ultimis terminis collatis.

(2) $l \sqcap \dfrac{4afga + 4ae\beta g + 2ef\beta^2 + 2af^2\beta}{\beta a^2}$ ex penultimis collatis.

(3) $$h \sqcap \frac{\begin{matrix}\boxed{4a^2eg\beta}\underset{//}{\boxed{+2a^2f^2\beta}} + 2\underset{///}{\boxed{6}}aef\beta^2 + 2e^2\beta^3 \\ -8fga^2 - 4\boxed{8}e\beta ga^2 \underset{///}{\boxed{-4ef\beta^2a}} - 2\underset{//}{\boxed{4}}f^2\beta a^2\end{matrix}}{\beta^2a^2}$$ ex antepenultimis collatis et destructis destruendis:

(4) $h \sqcap \dfrac{2aef\beta^2 + 2e^2\beta^3 - 8fga^3 - 4e\beta ga^2 - 2f^2\beta a^2}{\beta^2a^2}$.

Iam ex secundis collatis, fiet:

(5) $\beta^3e^2 \underset{//}{\boxed{+4aef\beta^2}}\boxed{+4e^2\beta^3} - 16fga^3 - 8e\beta ga^2 - 4f^2\beta a^2 \sqcap \underset{//}{\boxed{4ef\beta^2a}}\boxed{+4e^2\beta^3}$ et destructis destruendis, atque ordinando fiet:

(6) $e^2 - \dfrac{8ga^2}{\beta^2}\,e + \dfrac{16\boxed{64}g^2a^4}{\boxed{4}\beta^4} \sqcap \dfrac{16g^2a^4 + 16fga^3\beta + 4f^2\beta^2[a^2]}{\beta^4}$ et extracta radice:

(7) $\mp e \pm \dfrac{4ga^2}{\beta^2} \sqcap \dfrac{\sqrt{16g^2a^4 + 16fga^3\beta + 4f^2\beta^2[a^2]}}{\beta^2}$ sive

16–547,1 *a L ändert Hrsg. dreimal*

7 ex ultimis terminis collatis: Leibniz wertet die Gleichungen N. 38_{16} S. 535 Z. 6–13 aus.

(8) $e \sqcap \dfrac{+4ga^2 \mp \sqrt{16g^2a^4 + 16fga^3\beta + 4f^2\beta^2[a^2]}}{\beta^2}$.

Sed iam video non fuisse ad extrahendam radicem accedendum ante explicatam g, quia in ea est e, resumenda ergo est aequatio quinta, et in ea explicanda g, et g^2, est autem $g^2 \sqcap \dfrac{e^2\beta^4 + 2efa\beta^3 + f^2\beta^2a^2}{a^4}$ quare tota aequatio quinta per a^4 multiplicanda est, et fiet:

(9) $\boxed{\beta^3e^2a^4} + 24\boxed{16}fe\beta^2a^5 + 12\boxed{16}f^2\beta a^6 + 9\boxed{8}e^2\beta^3a^4 \boxed{+8e\beta^2fa^5}\boxed{-4f^2\beta a^6} \sqcap 0$.

destructisque destruendis et ordinando divisis prius omnibus per βa^4, fiet:

(10) $9e^2\beta^2 + 4af \frown 2 \frown 3e\beta + 16a^2f^2 \sqcap \boxed{16a^2f^2 - 12f^2a^2}\,4a^2f^2$ et extracta radice,

(11) $3e\beta + 2\boxed{4}af \sqcap \boxed{2af}$ sive

(12) $e \sqcap \dfrac{-2af}{\beta}$.

Ergo aequationi 12. iungendo aeq. 1. fiet:

(13) $g \sqcap \boxed{\dfrac{+2\not{a}f\beta - 3f\beta\not{a}}{3a^{\not{2}}}}\ \dfrac{-f\beta}{3a}$.

(14) Et eg, erit $\dfrac{+2f^2\beta}{9\beta}$ sive $\dfrac{2f^2}{9}$. quibus valoribus 12. 13. 14. in aequatione 2. substitutis, multiplicandus est totus valor ipsius aequationis 2. per $9\beta a$, ut omnia dividi possint per $[\beta a^2] \frown 9\beta a$. Nimirum omnes termini multiplicandi per 3, excepto uno, ubi est eg; omnes termini in quibus est e multiplicandi per $-6a^2f$, omnes termini in quibus est g, multiplicandi per $-3\beta^2f$, et eg per $2f^2a\beta$, fiet:

(15) $\dfrac{-12a^2f^2\beta^2 + 4a^2\beta^2f^2 - 12f^2\beta^2a^2 + 18a^2\beta^2f^2}{9\beta^2a^2} \sqcap l \sqcap \dfrac{-2f^2}{9a}$.

---

18 *Über der gestrichenen* 4 *von* $4a^2\beta^2f^2$: Error. Debet esse 8.
Über dem durch + *ersetzten Minuszeichen von* $-2f^2$: Debet esse +. Ergo in seqq. erratum.

1 f. e $\sqcap$ ... *(1)* et pro g substituendo eius valorem, item pro g^2 qui est: $+e^2\beta^4 + 2ef$ *(2)*. Sed *L*
15 βa *L ändert Hrsg.*

Eodem modo quaeremus valorem ipsius h ex aequatione 5^{ta}, ubi cum ingrediatur et $e^2 \sqcap \dfrac{4a^2f^2}{9\beta^2}$, ideoque multiplicanda sunt omnia per $9\beta^2 a$, et fiet:

$$(16)\ h \sqcap \frac{-2af\beta^2 2af3a\beta + 2\beta^3 4a^2f^2a + 8fa^3f\beta 3\beta^2 - 4\beta a^2 2f^2\beta^2 a - 2f^2\beta a^2 9\beta^2 a}{9\beta^4 a^3}$$

$$\sqcap \frac{-2f^2}{3\beta}.$$

Valores ergo ipsarum e, g, l, h. invenimus sane simplicissimos, nempe

$$e \sqcap \frac{-2af}{3\beta} \qquad g \sqcap \frac{-f\beta}{3a} \qquad l \sqcap \frac{-2f^2}{9a} \qquad h \sqcap \frac{-2f^2}{3\beta}.$$

Hoc ergo sucessu animati ad reliquas aequationes collatitias progrediamur, ubi collatis primis nominatorum terminis, fiet:

(17) $b \sqcap \dfrac{\ddagger e^2a + e\beta c}{\beta f}$ sive explicata e, fiet: $\dfrac{\ddagger 4a^2f^2a - 2af\beta c 3\beta}{9\beta^3 f}$, sive

(18) $b \sqcap \dfrac{\ddagger 4a^3f - 6a\beta^2 c}{9\beta^3}$ collatis porro secundis, fiet:

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3 *Kontrollrechnung:*

$$\begin{array}{r} 8 \\ \underline{24} \\ \underline{32} \\ -12 \\ -8 \\ \underline{-18} \\ \underline{-38} \\ -6 \end{array}$$

7 f. collatis ... fiet: *erg. L*

547,18–552,6 (15): Im zweiten Term des Zählers unterläuft Leibniz ein Koeffizientenfehler, der die weitere Rechnung beeinträchtigt. Konsequent gerechnet müßte in Gleichung (19) vor dem letzten Term der linken Seite $\ddagger$ stehen; in Gleichung (23) fehlt in den letzten beiden Termen der linken Seite der Faktor 9. In Gleichung (26) steht im zweiten Term der zweiten Zeile der Faktor 7 statt 4. Leibniz bemerkt schließlich den ersten Fehler, markiert den beeinträchtigten Bereich und setzt in S. 552 Z. 8 neu an.

(19) $\mp 2e\beta d9\beta^3 a \mp e\beta^2 c9\beta^3 + 2\beta g4a^3 fa \mp 2\beta g6a\beta^2 ca + \beta^2 f4a^3 f - \beta^2 f6a\beta^2 c \sqcap -2f^2a^2 3\beta^2 a$ sive explicando e, per 12, fiet

(20) $\pm 4afd3\beta^3 a \pm 2af\beta c3\beta^3 - \dfrac{2\beta 4a^3 f^2\beta \pm 2\beta 6\beta^2 caf\beta}{3} + \beta^2 f4a^3 f - \beta^2 f6a\beta^2 c \sqcap - 2f^2a^2 3\beta^2 a$ sive multiplicatis omnibus per 3, et divisis per $f\beta^2 a$, fiet:

(21) $\pm 36d\beta a \pm 30\boxed{18}\beta^2 c\ \boxed{-8a^2 f}\ \boxed{\pm 12\beta^2 c}\ \boxed{+4\boxed{12}a^2 f} - 18\beta^2 c \sqcap -22\boxed{18}fa^2$, ac destructis destruendis, fiet, $c \sqcap \dfrac{-22fa^2 \mp 36d\beta a}{\pm 30\beta^2 - 18\beta^2}$ sive

(22) $c \sqcap \dfrac{11fa^2 \pm 18d\beta a}{\mp 15\beta^2 + 9\beta^2}$.

Superest ut veniamus ad aequationem collatitiam ultimam, quam cum ingrediantur, e, b, c, ideoque multiplicanda erit tota illa aequatio per $\mp 135\beta^3 + 81\beta^3$, itaque fiet:

(23) $-\beta^2 d45\beta^2 2af \pm \beta^2 d27\beta^2 2af + f\beta da135\beta^3 \mp f\beta da81\beta^3, \mp\beta^2 g15, 4a^3 f + \beta^2 g15, 6a\beta^2 c + \beta^2 g9, 4a^3 f \mp \beta^2 g9, 6a\beta^2 c \pm \beta ga11fa^2\beta + \beta ga18d\beta a\beta \sqcap \pm 2f^2a^2 15\beta^3 - 2f^2a^2 9\beta^3$.

Pro g, et c, substituamus eorum valorem, et tota aequatio 23, multiplicanda erit per $\mp 15\beta^2 a + 9\beta^2 a$, sed prius dividemus omnia per βa, itaque sufficit multiplicari omnia per $\mp 15 + 9$, et fiet:

(24) $\pm 15\beta^2 d45\beta^2 2af - 9\beta^2 d45\beta^2 2af - 15\beta^2 d27\beta 2af \pm 9\beta^2 d27\beta 2af \mp 15f\beta da135\beta^3 + 9f\beta da135\beta^3 + 15f\beta da81\beta^3 \mp 9f\beta da81\beta^3,, -5f\beta\beta^2 15, 4a^2 f \pm 3f\beta\beta^2 15, 4a^2 f,, - 3\beta^2 f\beta 15, 6, 11fa^2 \mp 3\beta^2 f\beta 15, 6, 18d\beta a,, \pm 5\beta^2\beta f9, 4a^2 f - 3\beta^2\beta f9, 4a^2 f,, \pm 3\beta^2 f\beta 9, 6, 11fa^2 + 3\beta^2 f\beta 9, 6, 18d\beta a,, +5f\beta\beta^2 11, 4a^2 f \mp 3f\beta\beta^2 11, 4a^2 f,, \pm 5\beta^2 f18d\beta^2 a - 3\beta^2 f18d\beta^2 a \sqcap -30f^2a^2 15\beta^3 \pm 18f^2a^2 15\beta^3 \pm 30f^2a^2 9\beta^3 - 18f^2a^2 9\beta^3$.

Sed ut error, qui in calculum tam prolixum, quam est postremus, irrepere potuit, vitetur; tandem aequationem 23 aliter enuntiabimus:

4 – $2f^2a^2 3\beta^2 a$ *(1)* adeoque (21) c $\sqcap$ $18f^{\cancel{2}}a^{\cancel{3}2}\cancel{\beta^2} \mp 36\cancel{f}da^{\cancel{2}}\beta^{\cancel{3}} - 8\cancel{\beta^2}a^{\cancel{3}2}f^{\cancel{2}} \pm 12\beta^{\cancel{4}2}c\cancel{a}\cancel{f} + 12\cancel{\beta^2}f^{\cancel{2}}a^3 - 18\beta^{\cancel{4}2}\cancel{f}\cancel{a}c \sqcap -18f^{\cancel{2}}a^{\cancel{3}2}\cancel{\beta^2}$, explicata g. divisisque omnibus per $fa^2\beta^2$, hunc denique habebimus valorem ipsius c, (22) c $\sqcap$ *(2)* sive *L* 14 $\mp 15\beta^2 a + 9\beta^2 a$, *(1)* et fiet: $\pm\beta^2 d45\beta^2 2af15\beta^2 a - \beta^2 d45\beta^2 2af9\beta^2 a - \beta^2 d27\beta^2 2af15\beta^2 a \pm \beta^2 d27\beta^2 2af9\beta^2 a$ *(2)* sed *L*

∓ 49

(25) $\boxed{-45,2}\,\beta^4 daf$ $\boxed{\mp 15,4}\,\beta^2 a^{\not\beta 2} fg$ $\boxed{\begin{matrix}+15\\ \mp\ 9\end{matrix}}$, $6\beta^{\not 4\,2}\not d gc$ $+18\beta^3 da^{\not 2}g \sqcap \pm 2,15f^2a^2\beta^2$

$\boxed{\pm 27,2}$ . . . . $+\ 9,4$ . . . . , 6 . . . . $-2,\ 9$ . . .

$+\boxed{135}45$ . . . . $\boxed{\pm 11}$ . . . . ∓ 1 $\qquad \dfrac{-f\beta}{3\not d}$

$\mp\boxed{81}27$ . . . .

$\dfrac{-f\beta}{3\not d}$ $\qquad \dfrac{-f\beta}{3\not d}$

$$\frac{11fa^2 \pm 18d\beta a}{\boxed{\mp 15\beta^2 + 9\beta^2}}$$

sive (26) $3,+\,45\ \beta^{\not 4}d\not d\not f$ $\qquad \pm\ \ 49\ \not\beta^{\not 3}a^{\not 2}f^{\not 2}$ $\qquad \sqcap 0.$

$3,\mp\ 27$ . . . . $\qquad -\ 9,7$ . . .

$6,+\ 18$ . . . . $\qquad \pm 6,11$ . . .

-18 . . . . $\qquad \mp 6,15$ . . .

$\qquad +\ 6,9$ . . .

sive (27) $\begin{matrix}+225\beta d\\ \mp\ 81\,..\end{matrix} \sqcap \begin{matrix}\mp 25af,\\ +\ 9\,..\end{matrix}$ $\qquad$ sive (28) $d \sqcap \dfrac{\begin{matrix}\mp\ 25af\\ +\ \ 9\,..\end{matrix}}{\begin{matrix}+225\beta d\\ \mp\ \ 81\,..\end{matrix}}$.

Habita d, facile habetur c, fiet enim ex aeq. 22 iuncta 28

$$(29)\ c \sqcap \frac{11,\overset{25}{\cancel{225}},fa^2 \mp 11,\overset{9}{\cancel{81}},fa^2 - \overset{2}{\cancel{18}},25a^2f \pm 18,\cancel{9}a^2f}{\mp 15,\underset{25}{\cancel{225}}\beta^2 + 15,\underset{9}{\cancel{81}}\beta^2 + \cancel{9},225\beta^2 \mp \cancel{9},81\beta^2}$$

8–12 *Nebenrechnungen:*

18	45	∓ 27	∓ 15	$\pm\ 49$	-63
5	3	3	6	.. 66	$+54$
90	135	∓ 81	∓ 90	± 115	$-\ 9$
	90			$\mp\ 90$	
	225			$\pm\ 25$	

$$\text{sive (30) } c \sqcap \frac{\begin{array}{l} +9,\ \ 25fa^2 \\ \text{ǂ}\ \ \ \ \ \ 81\ \ldots \end{array}}{\begin{array}{rl} & \text{ǂ}3 \frown 125\beta^2 \\ & \ \ 27\ \ldots \\ +3 \frown & \ \ 45\ \ldots \\ & \ \ 75\ \ldots \end{array}} \sqcap \frac{\begin{array}{l} +3 \frown 25fa^2 \\ \text{ǂ}3 \frown\ \ 9\ \ldots \end{array}}{\begin{array}{l} \text{ǂ}\ 125\,\beta^2 \\ \ldots\ 27\ \ldots \\ +\ \ 45\ \ldots \\ \ldots\ 75\ \ldots \end{array}} \sqcap \frac{\begin{array}{l} +3 \frown 25fa^2 \\ \text{ǂ}\ \ldots\ \ 9\ \ldots \end{array}}{\begin{array}{l} \text{ǂ}\ [152]\,\beta^2 \\ +\ 120\ \ldots \end{array}}\ .$$

$$\text{Ac denique erit (31) } b \sqcap \frac{\begin{array}{ll} -4,125a^3f & -\ 6,3,25a^3f \\ \ldots\ \ldots\ \ 27\ \ldots & \text{‡}\ \ldots\ \ldots\ \ 9\ \ldots \\ \text{‡}\ \ldots\ \ 45\ \ldots & \\ \ldots\ \ldots\ \ 75\ \ldots & \end{array}}{\begin{array}{l} \text{ǂ}\ 9,125\beta^3 \\ \ldots\ \ldots\ \ 27\ \ldots \\ +\ \ldots\ \ 45\ \ldots \\ \ldots\ \ldots\ \ 75\ \ldots \end{array}}\ , \text{seu } b \sqcap \frac{\begin{array}{l} \text{‡}\ 642fa^3 \\ -\ 602\ \ldots \end{array}}{\begin{array}{l} \text{ǂ}1368\beta^3 \\ +1080\ \ldots \end{array}}\ .$$

550,15–551,5 *Nebenrechnungen:*

zu Z. 15:

4		15 ⌢ 25
~~2~~ ~~2~~5	~~+~~ 25	
~~9~~9		9 ⌢ 9

zu Z. 1–5:

+9, 25	ǂ99	ǂ3 ⌢ 125	9 ⌢ 15
	<u>‡18</u>	<u>ǂ3 ⌢ 27</u>	<u>9 ⌢ 25</u>
	ǂ81	3 ⌢ ǂ152	3 ⌢ 45
			3 ⌢ 75

6–12 *Nebenrechnungen:*

	‡45	125	18	152	45
	<u>‡75</u>	<u>27</u>	<u>25</u>	<u>9</u>	<u>75</u>
‡18	120	152	90	1368	120
<u>9</u>	<u>⌢ 4</u>	<u>450</u>	<u>36</u>		<u>8</u>
‡162	‡480	−602	450		1080
	<u>‡162</u>				
	‡642				

2+552,5 125 *L ändert Hrsg. zweimal*

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9 (31): Leibniz vergißt den Faktor 4 aus den ersten zwei Summanden im Zähler des Wertes von b, wie die zugehörige Nebenrechnung zeigt. Der Fehler überträgt sich auf die quadratische Gleichung in y im Zähler des Bruches S. 552 Z. 4–6.

Inventis ergo tandem valoribus omnium incognitarum ex quibus primario quaesitae sunt, b, c, d, e, $\boxed{f}$, g; f autem pro arbitrio sumi potest, cum qualiscunque supponatur evanescat; fractionem quaesitam $\dfrac{\frac{b}{a}y^2+cy+da}{\frac{e}{a}y^2+fy+ga}$ habebimus inventam, fiet enim:

$$\frac{\dfrac{\begin{array}{l}\text{‡}\ 642fa^2\\ -\ 602\ldots\end{array}}{\text{ǂ}1368\beta^3}\,y^2+\dfrac{\begin{array}{l}+3,25fa^2\\ \text{ǂ}\ ..\ 9\ \ldots\end{array}}{\text{ǂ}\,[152]\,\beta^2}\,y+\dfrac{\begin{array}{l}\text{ǂ}\,25fa^2\\ +\ 9\ldots\end{array}}{+225\,\beta}\\ \quad +1080\,..\qquad\qquad +\ 120\ ..\qquad\qquad \text{ǂ}\ \ 81\,..}{\dfrac{-2f}{3\beta}\,y^2+\qquad f\ \ y\qquad \dfrac{-f\beta}{3}}.$$

[*2. Ansatz*]

(1) $g \sqcap \dfrac{-e\beta^2-f\beta a}{a^2}$.

(2) $l \sqcap \dfrac{+4fga^2+4e\beta ga+2ef\beta^2+2f^2\beta a}{\beta a^2}$.

(3) $h \sqcap \dfrac{\begin{array}{c}\boxed{4ega^2\beta}\ \boxed{2f^2a^2\beta}+2\boxed{6}ef\beta^2a+2e^2\beta^3\\ -8fga^3-4\boxed{8}e\beta ga^2\boxed{-4ef\beta^2a}-\boxed{4}2f^2\beta a^2\end{array}}{\beta^2a^2}$

(4 - 5 - 6 - 7 - 8) $\beta^3e^2\boxed{+4ef\beta^2a}\ \boxed{+4e^2\beta^3}-16fga^3-8e\beta ga^2-4f^2\beta a^2 \sqcap \boxed{+4e\beta^2af}\ \boxed{+4e^2\beta^3}$ et explicando g,

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6 *Darunter*: Error fuit in calculo.

8 *Darüber*: Calculus iste probus est.

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15 probus: s. u. Erl. zu S. 554 Z. 5 f.

(9) $\boxed{\beta^3 e^2} + 24\boxed{16}fe\beta^{\cancel{2}}a + 12\boxed{16}f^2\cancel{\beta}a^2 + 9\boxed{8}e^2\beta^{\cancel{3}2}\boxed{+8e\beta^2 fa}\ \boxed{-4f^2\beta a^2} \sqcap 0$. et ordinando ad extrahendam radicem,

(10) $9e^2\beta^2 + 4fa \frown 2, 3e\beta + 16f^2a^2 \sqcap \boxed{16f^2a^2 - 12f^2a^2}4f^2a^2$. Et extrahendo:

(11) $3e\beta + 2\boxed{4}af \sqcap \boxed{2af}$, sive, ut obtineamus tandem valorem ipsius e,

(12) $e \sqcap \dfrac{-2af}{3\beta}$. Eumque valorem substituendo in valorem ipsius g, fiet:

(13) $g \sqcap \dfrac{+2af\beta - 3af\beta}{3a^2} \sqcap \dfrac{-\beta f}{3a}$.

Hos valores ipsarum e, et g, repertos inseramus in valore ipsius h, aeq. 3. ubi reperitur et

(14) $eg \sqcap \dfrac{+2f^2}{g}$, insertis ergo valoribus fiet:

(15) $h \sqcap \dfrac{-2f\cancel{\beta^2}\cancel{a}2\cancel{a}f3\cancel{a}\cancel{\beta} + 2\cancel{\beta^3}4\cancel{a^2}f^2\cancel{a} + 8f\cancel{a^3}\cancel{\beta}f3\cancel{\beta^2} - 4\cancel{\beta}\cancel{a^2}2f^2\cancel{\beta^2}\cancel{a} - 2f^2\cancel{\beta}\cancel{a^2}9\cancel{\beta^2}\cancel{a}}{9\beta\cancel{^4}\cancel{a^2}}$

sive $h \sqcap \dfrac{-12\boxed{+8} + 24\boxed{-8} - 18}{9\beta}f^2 \sqcap \dfrac{-6f^2}{9\beta} \sqcap \dfrac{-2f^2}{3\beta}$. Ergo denique

(16) $h \sqcap \dfrac{-2f^2}{3\beta}$.

Nunc ad reliquas collationes pergendum est.

(17) $\ddagger 3\beta^2 c2af \ddagger b\beta f9\beta^2 \sqcap 4a^2f^2a$, et proinde (18) $b \sqcap \dfrac{\ddagger 4a^3f^{\cancel{2}} - 6\beta^2 ca\cancel{f}}{9\beta 3\cancel{f}}$.

(19) $[a^2\beta]\ l \sqcap \dfrac{-4fa\beta f}{3} + \dfrac{8f^2\beta a}{9} - \dfrac{4af^2\beta}{3} + 2f^2\beta a$ seu $[9a^2\beta]\ l \sqcap -24\boxed{12}f^2\beta a\boxed{+8f^2\beta a}\ \boxed{-12f^2\beta a} + 26\boxed{18}f^2\beta a$. Ergo $l \sqcap \dfrac{2f^2\beta a}{9\beta a^2} \sqcap \dfrac{2f^2}{9a}$.

15 (19) *(1)* $\ddagger 2\mathrm{d}2\mathrm{af}9\beta^2 \ddagger \beta\mathrm{cf}3\boxed{9}\beta^2 + 2\mathrm{g}4\mathrm{a}^3\mathrm{f} \dagger 2\mathrm{g}\beta^2\mathrm{ca}\boxed{+\mathrm{f}4\mathrm{a}^2\mathrm{f}\beta}\boxed{\dagger\beta\mathrm{f}6\beta^2\mathrm{c}} \sqcap$ *(a)* $-\mathrm{a}^2 2\mathrm{f}^2 9$ *(b)* $-\mathrm{a}^2 2\mathrm{f}^2 5\boxed{9}\beta$. $\ddagger 108\mathrm{da}\beta^{\cancel{2}}\cancel{\mathrm{f}} \ddagger 11\boxed{9}\beta^{\cancel{3}2}\mathrm{c}\cancel{\mathrm{f}}\boxed{+8\mathrm{a}^2\mathrm{f}\beta\mathrm{f}}\boxed{\ddagger 2\beta^2\mathrm{c}\beta\mathrm{f}} \sqcap -22\boxed{30}\mathrm{a}^2\mathrm{f}^{\cancel{2}}\beta$. Unde $\mathrm{c} \sqcap \dfrac{\dagger 22\mathrm{a}^2\mathrm{f} - 108\mathrm{da}\beta}{11\beta^2}$. Sed quia *(aa)* haec ra *(bb)* hic valor ipsius c dissentit ab eo quem superiore calculo inveni, ideo repetendus est calculus aeq. 18. *(2)* | $\mathrm{a}^2\beta$ *erg. Hrsg.* | $\mathrm{l} \sqcap$ *L* 16 $9\mathrm{a}^2\beta$ *erg. Hrsg.*

19 inveni: s. Gleichung (22) S. 549 Z. 7.

(20) $\mp 2e\beta da \mp e\beta^2 c \pm 2b\beta g\cancel{a} \pm b\beta^2 f \sqcap ha^3$

$\dfrac{-2af}{3\beta}$ $\quad \dfrac{-2af}{3\beta}$ $\quad \dfrac{-\beta f}{3\cancel{a}}$ $\quad \dfrac{-2f^2}{3\beta}$

(21) $\pm 4a^2 f\beta d \pm 2af\beta^2 c \mp 2b\cancel{\beta^3} f \pm 3b\cancel{\beta^3} f \sqcap -2f^2a^3$

$\dfrac{\pm 4a^3 f - 6\beta^2 ca}{9\cancel{\beta^3}}$

(22) $\pm 36a\cancel{^2}\cancel{f}\beta d \pm 12\,\boxed{18}\,\cancel{a}\cancel{f}\beta^2 c \boxed{+4a^3f^2} \boxed{\mp 6\beta^2 caf} \sqcap -22\,\boxed{18}\,f\cancel{^2}a\cancel{^3}{}^2$. Adeoque $c \sqcap \dfrac{\mp 11fa^2 - 36a\beta d}{12\beta^2}$.

Porro $\mp e\beta\cancel{^2}d \mp f\beta da \pm b\cancel{\beta^2}g \pm c\cancel{\beta}g\cancel{a} \sqcap la\cancel{^3}{}^2$.

$\dfrac{-2af}{3\cancel{\beta}}$ $\quad \dfrac{\pm 4a\cancel{^3}{}^2 f - 6\beta^2 c\cancel{a}}{9\cancel{\beta^3}}$ $\quad \dfrac{-\cancel{\beta} f}{3\cancel{a}}$ $\quad \dfrac{-\cancel{\beta} f}{3\cancel{a}}$ $\quad \dfrac{2f^2}{9\cancel{a}}$

$\dfrac{\mp 11fa^2 - 36a\beta d}{12\cancel{\beta^2}}$

(23) $\boxed{\pm 2, 3, 12af\beta d}\,\boxed{\mp 3\,\boxed{9}, 12af\beta d} - 4, 4a^2f^2 \pm 6, 4\beta^2 cf + 11, 3f^2a^2 \pm 2\,\boxed{3}, 36a\beta df \sqcap 2, 12f^2a^2$ sive

$\pm 24\beta^2 c\cancel{f} \pm 72a\beta d\cancel{f} \sqcap 7f\cancel{^2}a^2$, sive explicando c, fit
$-22fa^2 \boxed{\mp 72a\beta d \pm 72a\beta d} \sqcap 7fa^2$.

Quae aequatio est impossibilis. Unde consequitur exitum sic quidem reperiri non posse.

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5 f. Adeoque $c \sqcap$: Im ersten Term des Zählers steht irrtümlich 11 statt 22. Der Fehler pflanzt sich bis zum Ende der Rechnung fort.

Zum 3. Teil (S. 555)